UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2015/2016

TITLE OF THE PAPER: COMPUTATIONAL METHODS-I

COURSE NUMBER: P262

TIME ALLOWED:

SECTION A:	ONE HOUR
SECTION B:	TWO HOURS

INSTRUCTIONS:
THERE ARE TWO SECTIONS IN THIS PAPER:

- **SECTION A:** IS A WRITTEN PART. ANSWER THIS SECTION ON THE ANSWER BOOK. IT CARRIES A TOTAL OF **30** MARKS.
- **SECTION B:** IS A PRACTICAL PART WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. IT CARRIES A TOTAL OF **70** MARKS.

Answer **all** the questions from Section A and **all** the questions from Section B. Marks for different sections of each Question are shown in the right hand margin.

THE PAPER HAS 6 PAGES, INCLUDING THIS PAGE.

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Section A – Use a pen and paper to answer these questions

Question 1

(a) How can you convert 1/2 in Maple into a floating-point number?

[1 mark]

(b) What is the difference between I and i in Maple?

[1 mark]

(c) Translate the following expressions into Maple statements.

(i) $a^b \sin(x)$

(ii) $\frac{d^2x(t)}{dt^2} = -\beta\frac{dx(t)}{dt} + \sin(3x(t))$

(iii) $\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx$

(iv) $\frac{x\log(t)}{y+ze^{-bt}}$

(v) $\psi = \frac{4}{\pi}\sum_{n=0}^{N} \frac{1}{(2n+1)} \cos((2n+1)(x-ct))$

[5 marks]

(d) Given the following Maple procedure:

> v := proc(t)
if (t < 0) then
0;
elif (0 < t and t <= 10) then
9.8 t;*
else
9.8 t -t** 2;*
end if
end proc;

determine the output of the following statements

(i) > v(-0.1);

(ii) > v(0.5);

(iii) > v(10);

[3 marks]

Question 2

(a) Given that

$$\mathcal{A} = 1 + \frac{2}{3} + \frac{4}{5} + \frac{6}{7} + \frac{8}{9} + \frac{10}{11}$$

(i) Write a Maple program that uses the **sum** command to evaluate $\mathcal{A}$.

[2 marks]

(ii) Write a Maple program that uses the **for loop** command to evaluate $\mathcal{A}$.

[3 marks]

(b) A recursion formula for Legendre Polynomial is given by

$$P_{n+1}(x) = \frac{(2n+1)}{(n+1)} x P_n(x) - \frac{n}{(n+1)} P_{n-1}(x) \text{ for n =1, 2, 3.......}$$

Given that $P_0(x) = 1$, $P_1(x) = x$, write a pseudo-code to calculate $P_m(x)$ for any $m \geq 2$.

[3 marks]

(c) Write a pseudo-code to generate 10 data points (x_i, y_i) using the equation

$$y_i = 0.1 + 9.8x_i$$

for $x_i = i\Delta x$, i=1..10 with $\Delta x = 0.1$.

[2 marks]

Question 3

(a) **A circuit of resistors:** Consider a long network of resistors, all the resistors have the same resistance. The arrangement is such that the voltages $V_1 \ldots V_N$ at the internal points in the circuit satisfy the equations:

$$\begin{aligned} 3V_1 - V_2 - V_3 &= V_+, \\ -V_1 + 4V_2 - V_3 - V_4 &= V_+, \\ &\vdots \\ -V_{i-2} - V_{i-1} + 4V_i - V_{i+1} - V_{i+2} &= 0, \\ &\vdots \\ -V_{N-3} - V_{N-2} + 4V_{N-1} - V_N &= 0, \\ -V_{N-2} - V_{N-1} + 3V_N &= 0. \end{aligned} \tag{1}$$

with $V_+ = 5$ Volts.

(i) Express the system of equations as a matrix equation in the form $\mathbf{A} \cdot \mathbf{V} = \mathbf{W}$. and the vector $\mathbf{W}$ for $N = 6$.

[2 marks]

(ii) Write a Maple program to solve for the values of the V_i. You may use *LinearSolve* function in Maple.

[3 marks]

(b) The steady state temperature distribution $T(x)$ along a particular thin bar of unit length is given by

$$\frac{d^2T(x)}{dx^2} = 10(T(x) - 20)$$

The initial conditions at $T(x=0), = 100$ and $T'(x=0) = 1.0$. Using the maple commands:

(i) Find the exact solution for $T(x)$.
Plot the solution for the interval $0 \leq x \leq 1$.

[2 marks]

(ii) Solve the equation using the default numerical method available in Maple.
Plot the solution in the interval $0 \leq x \leq 1$.

[3 marks]

Section B – Practical Part

Question 4

(a) **System of equations:-** For a certain direct current (dc) circuits, Kirchhoff loop rules give the following 4 simultaneous current equation:

$$
\begin{aligned}
2x_1 + 3x_2 - 6x_3 - 5x_4 &= 0 \\
x_1 - 2x_2 - 4x_3 + 2x_4 &= 0 \\
-3x_1 + 2.5x_2 + x_4 &= 5 \\
x_2 - 23x_3 + 9.3x_4 &= -7.2
\end{aligned}
$$

Determine the values of x_1, x_2, x_3, and x_4.

[5 marks]

(b) The Legendre polynomials $P_n(x)$ are defined by the relation

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}((x^2-1)^n)$$

(i) Plot the first 6 Legendre polynomials (n = 0, 1, 2....5) in a single graph for x = -1..1.

[10 marks]

(ii) Find the roots of $P_3(x)$ i.e., solve the equation $P_3(x) = 0$

[3 marks]

(iii) What is the value of $P_3(x = 0)$?

[2 marks]

Question 5

Competition for the same food supply. Two biological species competing for the same food supply are described by the following population equation

$$\begin{aligned} \frac{dN_1(t)}{dt} &= (4 - 0.0002N_1(t) - 0.0004N_2(t))N_1(t) \\ \frac{dN_2(t)}{dt} &= (2 - 0.00015N_1(t) - 0.00005N_2(t))N_2(t) \end{aligned} \qquad (2)$$

(a) Find and identify the stationary points of the system.

[10 marks]

(b) Suppose that $N_1(0) = 8000$ and $N_2(0) = 4000$. Using the fourth-order RungeKutta method, calculate the numerical solutions of the above simultaneous equations. Taking $t = 0...25$, plot $N_1(t)$, $N_2(t)$ versus time on the same graph.

[10 marks]

(c) Plot also $N_1(t)$ versus $N_2(t)$. Discuss the results.

[10 marks]

Question 6

The intensity distribution of radiation from a blackbody at a temperature T, may be represented by a formula

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^3} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}$$

where λ wavelength in metres, T is the temperature in Kelvin, $c = 3 \times 10^8 \text{ms}^{-1}$, $h = 6.63 \times 10^{-34}\text{Js}$ and $k_B = 1.38 \times 10^{-23}\text{JK}^{-1}$.

(a) Plot the following on one graph $I(\lambda)$ at temperature $T = 2000\text{K}$ over

(i) the visible wavelength range $0.4\mu\text{m}$ to $0.7\mu\text{m}$ in blue solid line and

(ii) wavelength range $0.3\mu\text{m}$ to $10\mu\text{m}$ in a red dotted line.

[15 marks]

(b) Determine the wavelength λ_m at which the intensity is maximum.

[5 marks]