UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2015/16

TITLE O F PAPER: MODERN PHYSICS & WAVE OPTICS

COURSE NUMBER: P231

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SEVEN PAGES INCLUDING THE COVER PAGE

THE LAST PAGE CONTAINS INFORMATION THAT MAY BE USEFUL IN SOME PROBLEMS

SOME SOLUTIONS ARE INCOMPLETE WITHOUT DIAGRAMS THAT ILLUSTRATE HOW THE SOLUTION WAS OBTAINED

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QUESTION 1

(a) Discuss what conditions must be met by

i. the path difference, and **(2 marks)**

ii. the phase angle **(2 marks)**

for two sinusoidal beams to interfere constructively?

(b) Two slits are separated by a distance of $d = 0.120$ mm and are illuminated with light of wavelength 500 nm, and the screen on which the interference patter is observed is placed a distance $L = 1.20$ m. Calculate the phase angle between the two wave fronts arriving at a point P which is a vertical distance y from the axis when

i. $\theta = 0.500°$ and **(3 marks)**

ii. $y = 5.00$ mm. **(3 marks)**

iii. What is the value of θ for which the phase difference is 0.333 rad? **(3 marks)**

iv. What is the value of θ for which the path difference is $\lambda/4$. **(3 marks)**

(c) A thin film of refractive index 1.58 is applied on a glass lens of refractive index 1.67 to a thickness of 400 nm. Light travels from air towards the lens.

i. Explain what happens to the phase angle of the light reflected from the film and from the glass. **(3 marks)**

ii. Determine the wavelengths that are strongly reflected when white light strikes the film at normal incidence. **(6 marks)**

QUESTION 2

(a) A screen is placed 50.0 cm from a single slit which is illuminated with a 690 nm light. If the distance between the first and third minima in the diffraction pattern is 3.00 mm, what is the width of the slit? **(5 marks)**

(b) A painting was made with a very high number of dots of a diameter of about 2.00 mm and close to each with. The colours of the dots were red and green. Outside what distance would you be unable to discern individual dots on the painting? Assume that $\lambda =$ 500 nm, and that your pupil diameter is 4.00 mm. **(3 marks)**

(c) A beam of light of intensity I_0 linearly polarized along the z-axis travels along the y-axis towards a polarizing filter, which is in the x-z plane with its axis aligned at an angle of 45.0° with the z-axis. A second polarizing filter is placed in front of the first at an angle of 45.0° with the first.

  i. Find the intensity of the beam emerging from the second filter in terms of the incident beam. **(2 marks)**

  ii. Compare the intensity and polarization of the incident and final beams. **(2 marks)**

(d) Draw a fully labeled schematic that illustrates the relationship between the velocities of the O-ray and E-ray in a birefringent (double refracting) crystal. **(4 marks)**

(e) The argon spectrum has a red line at 656.3 nm and a blue line at 434.0 nm is observed with a grating of 6000 grooves per cm.

  i. What are the angular separations between these two spectral lines in the first order? **(4 marks)**

  ii. At what angle does each of the two lines appear in the second order spectrum? Comment on your solution comparing it to that in i. **(2 marks)**

  iii. At what angle does each of the two lines appear in the third order spectrum? Comment on your solution. **(3 marks)**

QUESTION 3

(a) What two assumptions did Planck make in dealing with the problem of Blackbody radiation? **(2 marks)**

(b) Using what you learned from the photoelectric effect explain what is more likely to cause sunburn in human skin between ultraviolet and infrared light of the same intensity. **(3 marks)**

(c) Electrons are ejected from a metallic surface with speeds ranging up to a maximum of 4.60×10^5 m/s when light of 625 nm is used.

  i. What is the work function of the metal in eV? **(4 marks)**

  ii. What are the cutoff frequency and cutoff wavelength for the metal? **(3 marks)**

  iii. What is the stopping potential for the metal? **(2 marks)**

(d) As a result of his observations, what assumptions did Compton make in his derivation of the Compton Shift equation. **(2 marks)**

(e) A 0.110 nm photon collides with a stationary electron. After the collision, the electron moves forward and the photon recoils backward (angle $\theta = 180°$). Find

  i. the kinetic energy, and **(5 marks)**

  ii. the momentum of the electron. **(4 marks)**

QUESTION 4

(a) An electron and a photon both have energy of 3.00 eV. Find the wavelength of

i. the electron and **(3 marks)**

ii. the photon, and **(3 marks)**

iii. comment on the results. **(1 mark)**

(b) Consider a proton confined to move within the nucleus with a diameter of 10^{-15} m.

i. Show that the kinetic energy K of a nonrelativistic particle of momentum p and mass m can be written as $K = \frac{p^2}{2m}$. **(2 marks)**

ii. Use the information in i. and the uncertainty principle to find the minimum kinetic energy of a proton confined in the said nucleus. **(5 marks)**

(c) Consider the earth-sun system where the centripetal force on the earth is provided by the gravitational force of the sun. Data: $G = 6.67 \times 10^{-11}\mathrm{Nm^2/kg^2}$, mass of the earth $M_E = 5.98 \times 10^{24}$ kg, mass of the sun $M_S = 1.99 \times 10^{30}$ kg, and average radius of orbit of the earth around the sun $r = 1.496 \times 10^{11}$ m.

i. Apply the Bohr hydrogen model to the earth-sun system, to show that the allowed radii of orbit for the earth are given by

$$r_n = \frac{n^2\hbar^2}{GM_SM_e{}^2},$$

where n is an integer quantum number. **(4 marks)**

ii. Calculate the numerical value of n for the known earth orbit. **(3 marks)**

iii. Find the difference between the orbits r_n and r_{n+1} and comment on the result obtained. **(4 marks)**

QUESTION 5

(a) What are the particles emitted through natural radioactivity and how do they differ from each other? **(3 marks)**

(b) Two nuclei having the same mass number are known as isobars.

i. Calculate the binding energy per nucleon for each of the isobars $^{23}_{11}\text{Na}$ and $^{23}_{12}Mg$. **(6 marks)**

ii. Since these isobars have the same number of nucleons why is there a difference in the binding energies? **(2 marks)**

(c) A radioactive sample contains 5.45 μg of pure aluminium 28 which has a half-life of 2.44 min. Take the molecular mass of this isotope to be 28 g/mol.

i. What is the number of nuclei in the sample? **(2 marks)**

ii. Find the activity of the sample in becquerels initially. **(2 marks)**

iii. What is the number of remaining nuclei after four hours? **(2 marks)**

(d) Find the number of nucleons (mass number) and the number of protons for the element denoted X in the nuclear reaction shown next: Explain how you get the said values.
$\text{X} \rightarrow {}^{55}_{26}\text{Fe} + e^{+} + \nu.$ **(2 marks)**

(e) A rock sample contains traces parent elements of ^{238}U which undergoes several decays finally resulting in ^{206}Pb, after intermediate decay process with much shorter lifetimes and negligible amounts present in the rock. Careful analysis shows that the ratio of the amount of ^{238}U to ^{206}Pb is 1.64. The lifetime of ^{238}U to ^{206}Pb is 4.47×10^{9} years. Assume that the rock originally contained no lead and that the rate of formation of the lead is controlled by the half-life of ^{238}U. Determine the age of the rock. **(6 marks)**

SOME INFORMATION THAT MAY BE USEFUL IN SOME PROBLEMS

Avogadro's number $A = 6.02 \times 10^{23}$ particles per mole
Bohr radius $a_0 = 5.291\,772 \times 10^{-11}$ m
Boltzmann's constant, $k_B = 1.3801 \times 10^{-23}$ J/K
Compton wavelength $\lambda_C = \frac{h}{m_e C} = 0.002\,43$ nm
Coulomb constant $k_e = 8.987\,551\,788 \times 10^{23}9$ N.m^2/C^2
Radii of orbit for the hydrogen atom $r_n = n^2\, a_0$
Rydberg constant $R_H = 1.097\,373 \times 10^7$ m^{-1}.
Planck's constant, $h = 6.626\,075 \times 10^{-34}$ Js
$\hbar = 1.054\,572 \times 10^{-34}$ Js
$hc = 1.986\,447 \times 10^{-25}$ Jm
$2\pi hc^2 = 3.741\,859 \times 10^{-15}$ J·m^2·s^{-1}
Speed of light in vacuum, $c = 2.997\,924\,58 \times 10^8$ m/s
Stefan-Boltzmann Constant $\sigma = 335.669\,6 \times 10^{-8}$ W/(m^{2}K^2)
Wien's displacement law $\lambda_{max} = \frac{hc}{4.965 k_B T}$

Nuclear Data

Electron charge, $e = 1.602\,177\,33 \times 10^{-19}$ C
Electron mass, $m_e = 9.109\,389\,7 \times 10^{-31}$ kg = 0.000 548 6 u
Neutron, $m_n = 1.674\,928\,6 \times 10^{-27}$ kg = 1.008 665 u
Proton mass, $m_p = 1.672\,623 \times 10^{-27}$ kg = 1.007 276 u

1 atomic mass unit = 1 amu = 1 u = $1.660\,540\,2 \times 10^{-27}$ kg $\equiv$ 931.494 MeV rest mass energy
1 eV = $1.602\,177\,33 \times 10^{-19}$ J: 1 MeV = $1.602\,177\,33 \times 10^{-13}$ J
$T_{1/2}(^{14}C)$ = 5730 years
$T_{1/2}(^{238}U) = 4.47 \times 10^9$ years
Ratio of carbon 14 to carbon 12 in the atmosphere = $\frac{N(^{14}C)}{N(^{12}C)} = 1.2987 \times 10^{-12}$
1Curie (Ci) = 3.7×10^{10} Becquerel (Bq)
$r_0 = 1.2 \times 10^{-15}$ m
Deuterium (^2D) atomic mass = 2.014 102 u
Alpha particle (α) (^4He) atomic mass = 4.002 603 u
Hydrogen (^1H) atomic mass = 1.007 825 u
Hydrogen molecular mass = 1.007 94 u
Iron (^{56}Fe) atomic mass = 55.934 942 u
Molybdenum (^{94}Mo) mass = 93.905 088 u
Neodymium (^{144}Nd) atomic mass = 143.910 083 u
Oxygen ^{15}O atomic mass = 15 .003 065 u
Nitrogen ^{15}N atomic mass = 15.000 109 u
Radium ($^{226}_{88}Ra$) atomic mass = 226.025 403 u
Radon ($^{222}_{86}Rn$) atomic mass = 222.017 570 u
Ruthenium (^{98}Ru) atomic mass = 97.905 287 u
Tritium (^3T) atomic mass = 3.016 049 u

$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$