UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2015/2016

TITLE OF PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT PAGE WHEN NECESSARY.

THIS PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THIS PAGE UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Useful Mathematical Relations

Gradient Theorem

$$\int_{\vec{a}}^{\vec{b}} (\nabla f) \cdot d\vec{l} = f(\vec{b}) - f(\vec{a})$$

Divergence Theorem

$$\int \nabla \cdot \vec{A} d\tau = \oint \vec{A} \cdot d\vec{a}$$

Curl Theorem

$$\int (\nabla \times \vec{A}) \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{l}$$

Line and Volume Elements

Cartesian: $d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z}$, $d\tau = dxdydz$

Cylindrical: $d\vec{l} = ds\hat{s} + sd\phi\hat{\phi} + dz\hat{z}$, $d\tau = sdsd\phi dz$

Spherical: $d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$, $d\tau = r^2 \sin\theta drd\theta d\phi$

Gradient and Divergence in Spherical Coordinates

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Dirac Delta Function

$$\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$$

Question 1: ELECTROSTATICS ....

Four equal point charges of charge $-q$ are located on the yz plane. The charges are located at the points: $(0, a, 0)$, $(0, -a, 0)$, $(0, 0, a)$ and $(0, 0, -a)$. Another point charge of charge $+q$ is on the x axis at a distance b from the origin.

(a) What is the electric field at the point $(b, 0, 0)$ due to the other four charges? (6

(b) What is the force on $+q$ due to the other charges? (3

(c) What is the electrostatic potential at the point $(b, 0, 0)$? (5

(d) Use the potential to calculate the field and show that your results are consistent. (5

(e) How much energy will it cost to bring all the five charges from infinity to their respective positions? (6

Question 2: ELECTROSTATIC II..

Consider a capacitor made from two infinitely long conductors with coaxial cylindrical surfaces. The inner cylinder of radius $s = a$ is embedded in a cylindrical shell of inner radius $s = b$ and outer radius $s = c$. Let the positive charge be on the inner cylinder and the negative charge be on the outer cylindrical shell.

(a) Use Gauss's law to determine the electric field for a length L inside the inner cylinder, i.e in the region $s < a$. (6

(b) Use Gauss's law to determine the electric field in the region inbetween the cylinder and the cylindrical shell, i.e in the region $a < s < b$ (4

(c) Use Gauss's law to determine the electric field inside the cylindrical shell. (6

(d) What is the potential difference between the inner cylinder surface ($s = a$) and the inner surface of the cylindrical shell ($s = b$). (6

(e) Use the potential difference to deduce that the capacitance per unit length is given by (3

$$C/L = \frac{2\pi\epsilon_0}{\ln(b/a)}$$

Question 3: Magnetostatics ....

(a) State Maxwell's equations for time independent fields. (4

(b) Consider a finite segment of wire lying along the z axis. Let the wire be of length L with one end at z_1 and the other end at z_2.

i. If the current is in the positive z axis, in what direction is the induced magnetic field? (3

ii. Use the direction of the field to deduce the direction of the vector potential. (3

iii. Draw a diagram showing the wire and field point P a distance s from the wire. (2

iv. Determine the potential $\mathbf{A}$ at the point P. (8

v. Use the potential to determine the induced field. (5

Note

$$\int \frac{1}{\sqrt{a^2+x^2}}dx = \ln\left[2\left(x+\sqrt{a^2+x^2}\right)\right]$$

In cylindrical coordinates:

$$\nabla \times \mathbf{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{z}$$

Question 4: Magnetostatics II..

Let there be a current which induces a field $B = (\alpha x/y^2)\hat{x} + (\beta y/x^2)\hat{y} + f(x,y,z)\hat{z}$ where α and β are constants.

(a) What is the relation between magnetostatic potential **A** and the field **B**? (1)

(b) What is the divergence of a magnetostatic field **B**? (1)

(c) State the differential and integral form of Ampere's law. (2)

(d) Find the most general possible form for the function $f(x,y,z)$. (6)

(e) Find the current density **J**. (10)

(f) Use the continuity equation to verify that **J** corresponds to a steady current distribution. (5)

Question 5: Induction and Alternating Current Circuits..........................

(a) Give Faraday's three observations on electromagnetic induction. (6

(b) State the integral form of Faraday's law. (1

(c) Starting with the integral form of Faraday's law, use Stoke's theorem to deduce that the differential form of Faraday's law is (8

$$\nabla \cdot \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

(d) A square loop of wire, with sides of length a, lies in the first quadrant of the xy plane, with one corner at the origin. In this region there is a time-dependent magnetic field $\mathbf{B}(y,t) = ky^3t^2\hat{z}$, where k is a constant. Find the emf induced in the loop. (10