UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND EGINEERING

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2015/2016

TITLE O F PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS.

EACH QUESTION CARRIES 25 MARKS.

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN.

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE.

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QUESTION 1

(a) Derive the basic kinematic equation: $v^2 = {v_0}^2 + (x - x_0)$. **(5 marks)**

(b) A ball is dropped from rest at a height h above the ground. Directly from below, a second ball is thrown vertically upwards from the ground at the instant the first ball is released. Determine the initial speed of the second ball if the two balls have to meet at a height $h/2$ above ground, in terms of h and g. **(8 marks)**

(c) Use a clear diagram as an aid to illustrate how the position vector $\vec{r}$ is obtained in spherical coordinates, and write down the equation for this vector. **(6 marks)**

(d) Find the volume of a quarter of a hollow sphere of inner radius R_1 and outer radius R_2 starting with the volume element in spherical coordinates. **(6 marks)**

QUESTION 2

(a) A force F is applied to accelerate block m_3 to the right. Masses m_1 and m_2 a connected by a cord over a pulley as shown in Figure 1. All surfaces and the pulley are frictionless. Find the force F required to keep m_1 and m_2 motionless with respect to the cart, in terms of the masses and g. **(8 marks)**

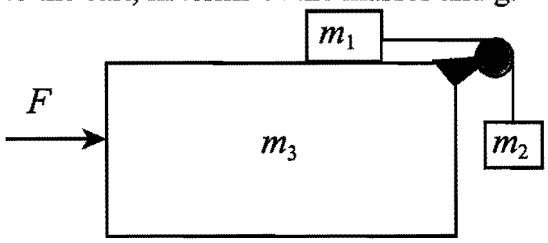

Figure 1.

(b) A and B are points on a fixed vertical shaft at a distance of $6l$ apart (see Figure 2). A particle of mass m is attached to A and B by light inextensible strings of length $5l$.

The particle moves in a horizontal circle with angular velocity $\omega = \sqrt{\frac{4g}{l}}$.

i. Make a correct resolved force diagram for the particle from which you can make useful equations. **(4 marks)**

ii. Determine the tensions T_1 (in upper string) and T_2 (in lower string). **(8 marks)**

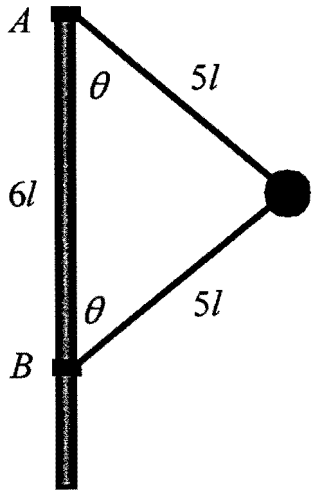

Figure 2.

(c) A 270 kg communication satellite is placed in a geostationary orbit 35,780 km above a relay Earth ground level. What is the speed of the satellite in orbit? **(5 marks)**

QUESTION 3

(a) Find the centre of mass of a uniform hemisphere of radius R with the centre of its base at the origin. **(9 marks)**

(b) An instrument carrying projectile explodes accidentally at the top of its trajectory. The horizontal distance between the launch point and the point of explosion is L. The projectile breaks into two pieces which initially fly apart horizontally. The larger piece has three times the mass of the smaller piece. The smaller piece follows the initial path but in the opposite direction and lands at the launch point. How far away does the larger piece lands in terms of L. **(9 marks)**

(c) An inverted garbage can of mass m is suspended in air by water from a burst pipe on the ground (see Figure 3). The water shoots up from the ground with a speed v_0, and at a constant rate $\frac{dm}{dt}$. Find the maximum height at which the garbage can rides. State assumptions made. **(7 marks)**

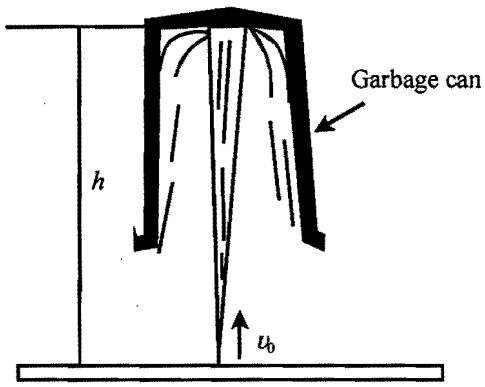

Figure 3.

QUESTION 4

(a) A small block slides from rest from the top of a frictionless sphere of radius R as shown in Figure 4. How far below the top of the sphere x does it lose contact with the sphere? **(7 marks)**

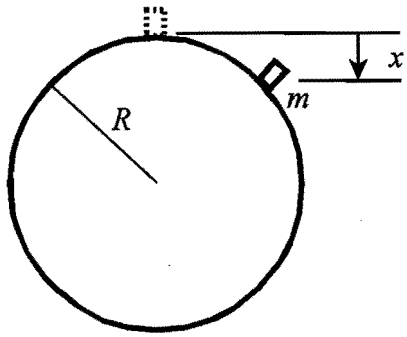

Figure 4.

(b) A body is projected upward with a velocity v_0 from the earth's surface (at radius $r = R_E$) where the gravitational force is given by $F = -\frac{GM_E m}{r^2}$, where G is the universal gravitational constant, M_E the mass of earth, m the mass of the projected body and r the distance from the centre of the earth. Use wth work-energy theorem to find the maximum height reached by the body in terms of R_E, v_0, and g. **(9 marks)**

(c) A particle of effective mass m is acted upon by a force F under a potential $U(r) = -U_0 4r^2 + U_0 2r^4$, where $a = 4\ \text{m}^{-2}$ and $b = 2\ \text{m}^{-4}$.

i. Find the force acting on the particle. **(2 marks)**
ii. Fid the equilibrium point(s) for the particle. **(3 marks)**
iii. Determine the stability of the equilibrium point(s). **(2 marks)**
iv. Determine the frequency of small oscillations about the equilibrium point(s). **(2 marks)**

QUESTION 5

(a) Find the moment of inertia of uniform thin stick of mass M and length L through an axis at one end perpendicular to the stick. **(5 marks)**

(b) Derive an expression that relates torque to angular momentum. **(5 marks)**

(c) The mass in a conical pendulum moves in a circular path given by
$\vec{r} = l \sin\alpha \cos\omega t\, \hat{\imath} + l \sin\alpha \sin\omega t\, \hat{\jmath}$,
and the force acting on the mass is
$\vec{F} = mg(-\hat{k})$. See Figure 5.

i. Find the velocity of the velocity of the mass. **(2 marks)**
ii. Find torque about A. **(3 marks)**
iii. Find the angular momentum about A. **(3 marks)**
iv. Find the torque about B. **(3 marks)**
v. Find the angular momentum about B. **(4 marks)**

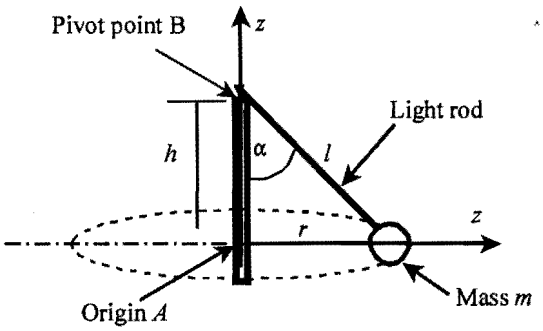

Figure 5.