UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE & ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2012/2013

TITLE OF PAPER	:	THERMODYNAMICS
COURSE NUMBER	:	P242/EE202
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS	:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS
		EACH QUESTION CARRIES 25 MARKS
		MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS 6 PAGES, INCLUDING THIS PAGE.

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QUESTION 1

(a) Fig. 1 shows a Carnot cycle which is based on a Carnot engine. Use the cycle to derive the equation below for the efficiency of a Carnot engine

$$\eta = 1 - \frac{T_C}{T_H},$$

where T_C and T_H represent the temperatures of the cold and hot reservoirs, respectively. (13 marks)

(b) The Otto cycle shown in Fig. 2 represents the operation of an internal combustion engine. The temperature at point '*a*' of the cycle is 57 °C whilst the pressure at the same point is 10^5 Pa. Due to compression of the gas in the engine, the pressure at point '*b*' increases from 10^5 Pa to 1.5×10^6 Pa and heat is generated in step '*b*' to '*c*' in such a way that the temperature rises to 2177 °C at point '*c*'.

(i) Find the temperatures at points '*b*' and '*d*'. (9 marks)

(ii) Calculate the thermal efficiency of the engine. (3 marks)

[**Hint**: Assume that the working medium of the engine is one mole of an ideal gas and that C_P and C_V are 29.20 $Jmol^{-1}K^{-1}$ and 20.88 $Jmol^{-1}K^{-1}$, respectively].

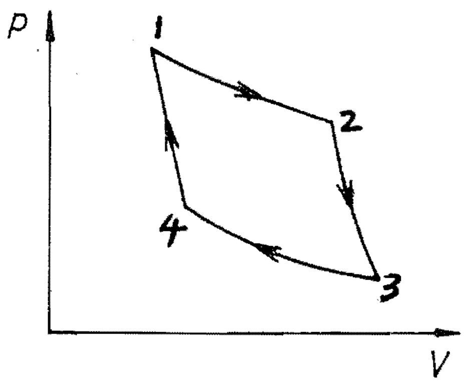

Fig. 1

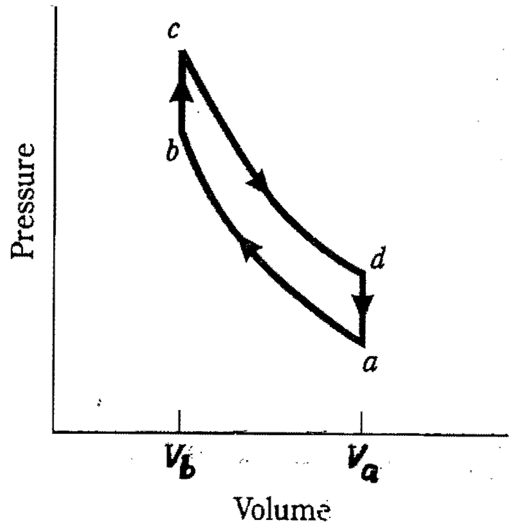

Fig. 2

QUESTION 2

(a) State the meaning of the word 'absorptivity'. (1 mark)

(b) A spherical body 0.5 m in diameter emits radiation. The surface temperature of the body is 1200°C and its emissivity is 0.6. What will be the intensity of the radiation emitted by the body at a distance 10 m away from it? Assume that the temperature of the surroundings is negligible compared to the temperature at the surface of the body.

[Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$]. (5 marks)

(c) Show that the rate of heat flow through two slabs in series, having the same cross-sectional area A, but different thermal conductivities k_1 and k_2 and corresponding lengths L_1 and L_2, is given by

$$\frac{dQ}{dt} = \frac{A(T_1 - T_2)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}},$$

where T_1 and T_2 are the temperatures at the extreme ends of the two slabs ($T_1 > T_2$). (11 marks)

(d) A steel bar, 0.1 m long, is welded end to end to a copper bar 0.2 m long. Each bar has a square cross section, 0.02 m on a side. The free end of the steel bar is in contact with steam at 100 °C, and the free end of the copper bar is in contact with ice at 0°C. The thermal conductivities of steel and copper are 50.2 $Wm^{-1}K^{-1}$ and 385 $Wm^{-1}K^{-1}$, respectively. Assuming steady state conditions,

(i) calculate the rate of heat flow through the two bars. (4 marks)

(ii) calculate the temperature at the junction of the two bars. (4 marks)

QUESTION 3

(a) Explain the concept of "entropy". (1 mark)

(b) Suppose that 1 kg of water at 7°C is mixed with 2 kg of water at 37°C in a thermally insulated vessel. After equilibrium is reached, the mixture has a uniform temperature.

Find the total change in entropy of the system? (9 marks)

[Specific heat of water = 4190 J $kg^{-1}K^{-1}$].

(c) Imagine a hot reservoir at 400°C. The reservoir is used to evaporate 0.1 kg of water initially at 30°C.

(i) Find the minimum energy required to evaporate the water completely. (5 marks)

(ii) By how much does the entropy change after the evaporation? (6 marks)

(iii) Calculate the change in entropy of the reservoir? Assume that the temperature of the reservoir stays constant. (2 marks)

(iv) What will be the change in entropy of the universe?

[Latent heat of vapourisation = 2.26×10^6 Jkg^{-1}]. (2 marks)

QUESTION 4

(a) With the aid of the pV diagram shown in Fig. 3, discuss the principle of operation of a commercial refrigerator. (10 marks)

(b) With reference to the ideal cycle of a Carnot refrigerator shown in Fig. 4, derive an expression to show that the performance coefficient, ω of a Carnot refrigerator is given by

$$\omega = \frac{T_C}{T_H - T_C}$$

(10 marks)

(c) (i) Consider a refrigerator in a room at a temperature of 20 °C. If the temperature of the freezer of a refrigerator is -5 °C, how much electrical energy would be required to remove 700 kJ of heat from the freezer operating at its maximum coefficient of performance? (3 marks)

(ii) How much heat is deposited in the room by the refrigerator? (2 marks)

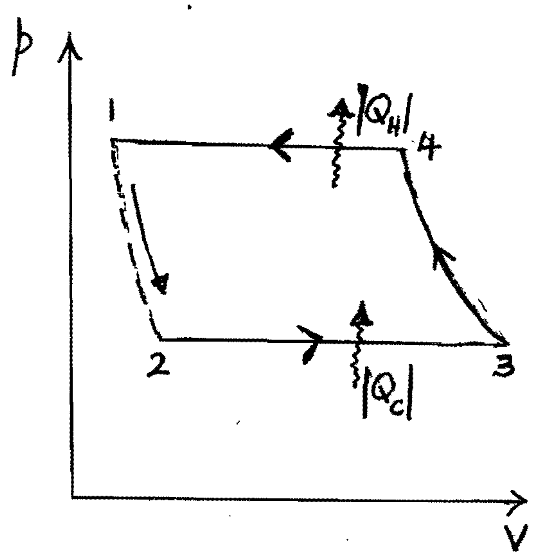

Fig. 3

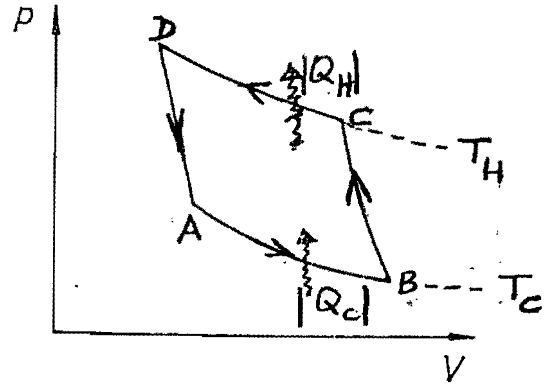

Fig. 4

QUESTION 5

(a) Derive an expression which shows the relationship between the average distance, λ, a molecule travels between collisions in a gas is given by

$$\lambda = \frac{1}{n\pi d^2\sqrt{2}}$$

where n is the concentration of molecules and d is the molecular diameter. Use appropriate diagram(s) for illustration. (15 marks)

(b) The molecular diameters of different kinds of gas molecules can be found experimentally by measuring the rates at which the molecules diffuse in the gas. The diameter d = 3.15×10^{-10} m has been reported for nitrogen.

Determine the mean-free-path of a nitrogen molecule at room temperature and at normal atmospheric pressure if the number of molecules per unit volume is 4.0×10^{22}. (2 marks)

(c) In general, the root-mean-square-speed of a gas molecule is given by the following relationship:

$$v_{rms} = \left(\overline{v^2}\right)^{\frac{1}{2}}$$

Show that v_{rms} depends on the temperature of the gas and its mass, as shown below.

$$v_{rms} = 1.73\left(\frac{kT}{m}\right)^{\frac{1}{2}}$$

(8 marks)

Hint:

$$\overline{v^2} = \frac{1}{N}\int_0^\infty N(v)v^2 dv$$

where

$$N(v) = 4\pi N\left(\frac{\lambda}{\pi}\right)^{\frac{3}{2}} v^2 \exp\left(-\lambda v^2\right)$$

$\lambda = m/2kT$ and the other symbols have their usual meanings.

$$\int_0^\infty v^4 \exp\left(-\lambda v^2\right)dv = \frac{3}{8}\left(\frac{\pi}{\lambda^5}\right)^{\frac{1}{2}}$$