UNIVERSITY OF SWAZILAND 179

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2010/2011

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: P461

TIME ALLOWED : THREE HOURS

THIS PAPER CONTAINS **FIVE** QUESTIONS. ANSWER ANY **FOUR** QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS

THIS PAPER CONTAINS **EIGHT** PAGES INCLUDING THE COVER PAGE.

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QUESTION ONE 180

(a) Explain the difference between a macrostate and a microstate of a system of particles. (2 marks)

(b) (i) Four coins marked a, b, c, and d are tossed. If the number of heads (H) and the number of tails (T) obtained in a toss define a macrostate,

(1) write down all the possible macrostates and (2 marks)
(2) calculate the number of microstates corresponding to each of the above macrostates. (4 marks)

(ii) Assuming the number of coins is increased from 4 to 8, what would be the maximum number of microstates, W_{max} that can be obtained in a toss. (2 marks)

(iii) What is the statistical significance of W_{max} for an assembly of particles, in general? (2 marks)

Hint: $W = \dfrac{N!}{\prod_s n_s!}$

(c) (i) What is meant by degeneracy of an energy level? (2 marks)

(ii) Find the degeneracy of an energy state having energy $E = kn^2$, k being a constant and $n^2 = n_x^{\,2} + n_y^{\,2} + n_z^{\,2} = 26$. n_x, n_y, n_z are quantum numbers corresponding to a quantum state. (4 marks)

(d) (i) What is meant by phase space? (2 marks)

(ii) Derive the following expression for the volume element in phase space, in terms of energy:

$$g(\varepsilon)d\varepsilon = 2\pi V(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$$

(5 marks)

QUESTION TWO 181

(a) Derive the Maxwell-Boltzmann distribution function for an isolated system of non-interacting particles in thermal equilibrium. (11 marks)

(b) Assuming that the constant β in the above distribution function is equal to -1/(kT), k being the Boltzmann constant, show that the entropy of the system

$$S = k \ln W$$

where W is the maximum thermodynamic probability of the system.

(6 marks)

(c) Given that the partition function of a system $Z = \sum_s g_s e^{\beta \varepsilon_s}$

show that the entropy of the system $S = Nk \ln Z + \frac{E}{T}$

(8 marks)

where the symbols have their usual meanings.

QUESTION THREE 182

(a) Two systems on non-interacting and identical particles in thermal equilibrium have entropies S_1 and S_2, and weights W_1 and W_2. If the two systems are combined, what is

(i) the total entropy (1 mark)

(ii) the total weight of the combined system? (1 mark)

Verify if your results agree with the equation S = k lnW (2 marks)

(b) Given that the density of states of a classical perfect gas is

$$g(\varepsilon)d\varepsilon = \frac{2\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon,$$

where the symbols have their usual meanings, show that the partition function of the system, $Z = \frac{V}{h^3}(2\pi mkT)^{3/2}$ (7 marks)

(c) (i) Show that the pressure of a classical gas, $P = NkT\left(\frac{\partial \ln Z}{\partial V}\right)_T$ (6 marks)

(Given: F = E - TS = - NkTlnZ)

(ii) Hence derive the ideal gas equation: $PV = NkT$ (4 marks)

(iii) Show that the total energy of the system, $E = \frac{3}{2}NkT$. (4 marks)

QUESTION FOUR

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(a) Obtain the mean energy of a quantum mechanical one-dimensional harmonic oscillator by setting up its partition function.

'High temperature is the classical limit of quantum mechanics'. Given that the mean energy of a classical oscillator is kT, verify this statement for the harmonic oscillator.

Given: for small vales of x, $e^x = 1 + \frac{x}{1!} + \frac{2}{2!}$

(12 marks)

(b) (i) State the basic assumptions of Einstein's theory of the specific heat of solids.

(ii) Obtain an expression for the specific heat of solids based on these assumptions.

(9 marks)

(c) At low temperatures the specific heat of a solid is proportional to T^3. Does Einstein's theory agree with this? Explain briefly.

(4 marks)

QUESTION FIVE 184

(a) Define Fermi energy. (2 marks)

(b) The Fermi function of a system is given as

$$f(\varepsilon) = \frac{1}{e^{(\varepsilon-\varepsilon_F)/kT} + 1}$$

(i) State the significance of this function as regards the distribution of the fermions in the system

(2 marks)

(ii) Compute the value of the Fermi function at 0 K for the cases:

$\epsilon < \epsilon_F$ and $\epsilon > \epsilon_F$ (2 +2 marks)

(iii) Sketch graphs of $f(\epsilon)$ versus ϵ for T = 0K and T > 0K and state the physical meaning that you can infer from them.

(4 marks)

(c) Use Fermi-Dirac statistics to show that the contribution by free electrons in a metal toward its specific heat is proportional to the absolute temperature.

Given: Fermi energy $\varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$

(8 marks)

(d) Calculate the Fermi energy of lithium having electron density of $5 \times 10^{28}\ m^{-3}$

(5 marks)

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty e^{-ax}\,dx = \frac{1}{a}, (a > 0)$$

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Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Planck's constant	h	6.63×10^{-34} J.s
Stefan - Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan constant	σ	5.67×10^{-8} $Wm^{-2}K^{-4}$
Atmospheric pressure		1.01×10^{5} Nm^{-2}
Mass of ${}_2^4$He atom		6.65×10^{-27} kg
Mass of ${}_2^3$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 L mol^{-1}