

UNIVERSITY OF SWAZILAND.

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FACULTY OF SCIENCE.

DEPARTMENT OF PHYSICS.

MAIN EXAMINATION 2010 / 2011

TITLE OF PAPER: SOLID STATE PHYSICS.

COURSE NUMBER: P 412.

TIME ALLOWED : THREE HOURS.

ANSWER ANY FOUR QUESTIONS . ALL CARRY EQUAL MARKS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE
INVIGILATOR.

Question One

- (a) Define the terms *lattice*, *primitive unit cell* and *conventional unit cell* of a crystal. (3 marks)
- (b) Draw **one** figure showing a conventional unit cell and also the primitive cell of an fcc lattice. (3 marks)
- (c) One side of a conventional unit cell of an f.c.c. lattice is 3 Å . What is the volume of its primitive unit cell? (2 marks)
- (d) Calculate the separation between two (123) planes of an orthorhombic lattice with cell lengths, $a = 0.82 \text{ nm}$, $b = 0.94 \text{ nm}$ and $c = 0.75 \text{ nm}$ in the x,y, z directions. (4 marks)
- (e) (i) What is meant by packing fraction of a crystal? (2 marks)
(ii) Determine the packing fraction of a bcc crystal (4 marks)
- (f) A first order reflection from the (111) planes of a cubic crystal was observed at a glancing angle of 11.2° when x-rays of wavelength 154 pm were used. Calculate the length of the side of each cell. ($1 \text{ pm} = 10^{-12} \text{ m}$) (3 marks)
- (g) In the X-ray photograph of a cubic lattice, lines are observed at the following Bragg angles in degrees: 6.6, 9.2, 11.4, 13.1, 14.7. Identify the lattice type. (4 marks)

Question Two

- (a) (i) State what is meant by *density of states* in terms of frequency ω as applied in lattice dynamics. (2 marks)

- (ii) By considering the boundary conditions on lattice waves in a cubic crystal, prove that for a three dimensional system, the density of states is given by the expression

$$D(\omega) = \frac{V\omega^2}{2\pi^2 v^3}$$

where V is the volume, v the velocity of sound. (8 marks)

- (b) (i) What is Debye approximation? (3 marks)

- (ii) Show that by applying Debye approximation, the density of states for a system with one atom per unit cell is given by

$$D(\omega) = \frac{9N\omega^2}{\omega_D^3}$$

where N is the number of atoms and ω_D , the Debye frequency. (7 marks)

- (c) Use the result in(b) above to show that the zero point energy of a lattice is given by

$$E = \frac{9N}{8} \hbar \omega_D$$

[Given: the mean energy of a harmonic of oscillator is

$$\bar{\epsilon} = \hbar \omega \left(\frac{1}{2} + \frac{1}{e^{\hbar \omega / kT} - 1} \right)] \quad (5 \text{ marks})$$

Question Three

- (a) State the assumptions Drude made in his *free electron theory* of metals. (3 marks)
- (b) Define the terms *mean free path* and *mobility* of an electron. (2+2 marks)
- (c) Show that according to Drude theory the d.c electrical conductivity of a metal can be expressed as:
- $$\sigma = \frac{ne^2\tau}{m}, \quad \text{where the symbols have their usual meanings.} \quad (10 \text{ marks})$$
- (d) (i) State *Wiedemann - Franz* law. (3marks)
- (ii) Write down the expression for *Lorenz number* and calculate its value. (2 + 3 marks)

Question Four

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- (a) (i) Define "Fermi energy". (2 marks)
- (ii) Derive an expression for the density of states of a system of electrons, given that the Fermi energy:

$$\epsilon_F = \frac{\hbar^2}{2m} \left(\frac{3\pi^2 N}{V} \right)^{2/3} \quad (6 \text{ marks})$$

- (iii) Calculate the density of energy states at 2.05 eV energy for a system of electrons in a volume of 1 cm³. (8 marks)
- (b) (i) Show that the electronic contribution to heat capacity of a metal is proportional to absolute temperature. (6 marks)
- (ii) Discuss the heat capacity of metals, explaining the difference, if any, between the above theory and experimental values. (3 marks)

Question Five

- (a) Using silicon as an example, explain how the electrical conductivity of a semiconductor can be increased by doping. (6 marks)
- (b) With the help of an appropriate diagram, derive an expression for the effective density of states in the conduction band of a semiconductor. Assume: $(\epsilon - \epsilon_F) \gg kT$. (10 marks)

Given:

Fermi -Dirac distribution function:
$$f(\epsilon) = \frac{1}{e^{(\epsilon - \epsilon_F)/kT} + 1}$$

Density of states for a system of fermions:

$$D(\epsilon)d\epsilon = \frac{4\pi}{h^3} (2m)^{3/2} \epsilon^{1/2} d\epsilon$$

- (c) A doped semiconductor has electron and hole concentrations $2 \times 10^{13} \text{ cm}^{-3}$ and $1.41 \times 10^{13} \text{ cm}^{-3}$ respectively. Calculate the electrical conductivity of the sample. (5 marks)
- [Take: $\mu_n = 4200 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$, $\mu_p = 2000 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$]
- (d) Discuss briefly the process of photoconductivity in semiconductors. (4 marks)

Appendix 1

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Various definite integrals.

$$\int_0^{\infty} e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}$$

$$\int_0^{\infty} e^{-ax^2} x dx = \frac{1}{2a}$$

$$\int_0^{\infty} e^{-ax^2} x^3 dx = \frac{1}{2a^2}$$

$$\int_0^{\infty} e^{-ax^2} x^2 dx = \frac{1}{4} \sqrt{\frac{\pi}{a^3}}$$

$$\int_0^{\infty} e^{-ax^2} x^4 dx = \frac{3}{8a^2} \left(\frac{\pi}{a} \right)^{1/2}$$

$$\int_0^{\infty} e^{-ax^2} x^5 dx = \frac{1}{a^3}$$

$$\int_0^{\infty} \frac{x^3 \cdot dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^{\infty} x^{1/2} e^{-\lambda x} dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^{\infty} \frac{x^4 e^x}{(e^x - 1)^2} dx = \frac{4\pi^4}{15}$$

$$\int_0^{\infty} \frac{x^{1/2}}{e^x - 1} dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2Physical Constants.

<i>Quantity</i>	<i>symbol</i>	<i>value</i>
Speed of light	c	$3.00 \times 10^8 \text{ ms}^{-1}$
Plank's constant	h	$6.63 \times 10^{-34} \text{ J.s}$
Boltzmann constant	k	$1.38 \times 10^{-23} \text{ JK}^{-1}$
Electronic charge	e	$1.61 \times 10^{-19} \text{ C}$
Mass of electron	m_e	$9.11 \times 10^{-31} \text{ kg}$
Mass of proton	m_p	$1.67 \times 10^{-27} \text{ kg}$
Gas constant	R	$8.31 \text{ J mol}^{-1} \text{ K}^{-1}$
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	$9.27 \times 10^{-24} \text{ JT}^{-1}$
Permeability of free space	μ_0	$4\pi \times 10^{-7} \text{ Hm}^{-1}$
Stefan constant	σ	$5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$
Atmospheric pressure		$1.01 \times 10^5 \text{ Nm}^{-2}$
Mass of ${}_2^4\text{He}$ atom		$6.65 \times 10^{-27} \text{ kg}$
Mass of ${}_2^3\text{He}$ atom		$5.11 \times 10^{-27} \text{ kg}$
Volume of an ideal gas at STP		22.4 L mol^{-1}