UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2010/2011

TITLE OF THE PAPER: CLASSICAL MECHANICS

COURSE NUMBER: P320

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 MARKS.
- MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS 6 PAGES, INCLUDING THIS PAGE.

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Question 1

(a) Write the Lagrangian and the corresponding equations of motion of a particle of mass m moving under a central conservative force.

[5 marks]

(b) Write down two conserved quantities obeyed by this particle. You may express the conservation laws using some mathematical formulas.

[4 marks]

(c) Derive the radial energy equation

$$\frac{1}{2}m\dot{r}^2 + \frac{l_z^2}{2mr^2} + U(r) = E$$

where E, l_z, and $U(r)$ are the total energy, angular momentum, and potential energy of the particle, respectively.

[5 marks]

(d) Describe the force that corresponds to the angular momentum term in the radial energy equation.

[3 marks]

(e) The last two terms in the radial energy equation can be combined to form an effective potential

$$U_{eff}(r) = \frac{l_z^2}{2mr^2} + U(r).$$

(i) Sketch $U_{eff}(r)$ against r with $U(r)$ replaced by an attractive potential like the gravitational potential.

[3 marks]

(ii) Draw on the diagram a value for E for which the motion is bounded. What shape is the orbit.

[3 marks]

(iii) Describe the motion of a particle which has $E = 0$.

[2 marks]

Question 2

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(a) Show that for a single particle with constant mass the equation of motion implies the following differential equation for the kinetic energy:

$$\frac{dT}{dt} = \mathbf{F} \cdot \mathbf{v}$$

while if the mass varies with time the corresponding equation is

$$\frac{d(mT)}{dt} = \mathbf{F} \cdot \mathbf{p}.$$

Here m, T, $\mathbf{p}$, and $\mathbf{v}$ correspond to the mass, kinetic energy, linear momentum and velocity of the particle, respectively. $\mathbf{F}$ is the external force acting upon the particle.

[6 marks]

(b) A simple pendulum consists of a mass m suspended from a fixed point by a weightless, extensionless rod of length l.

(i) Obtain the equation of motion and, in the approximation that $\sin\theta \approx \theta$, show that the natural frequency is $\omega_0 = \sqrt{g/l}$, where g is the gravitational field strength.

[3 marks]

(ii) Discuss the motion in the event that it takes place in a viscous medium with retarding force $2m\sqrt{gl}\dot{\theta}$.

[5 marks]

(c) Consider two particles of equal mass m. The force $\mathbf{F_2} = F_0\mathbf{e_x}$, where $\mathbf{e_x}$ is a unit vector along the x-axis acts on particle (2) and there is no force acting on particle (1). If the particles are initially at rest at the origin, what is the position, velocity and the acceleration of their center of mass?

[5 marks]

(d) The center of gravity of a system of particles is the point about which external gravitational forces exert no torque. For a uniform gravitational force, show that the center of gravity is identical to the center of mass for the system of particles.

[6 marks]

Question 3

122

(a) A tennis ball with mass m and diameter 0.1 m rolls down an inclined surface. Its center of gravity starts at height h above the laboratory bench. [The moment of inertia of a hollow sphere is $I = \frac{2}{3}mR^2$]

(i) What is the change in the height of the center of gravity of the tennis ball?

[3 marks]

(ii) What is the velocity of the tennis ball at the moment it first touches the bench?

[8 marks]

(iii) Carefully list all the assumptions you have made in your calculation.

[4 marks]

(b) Two spheres are of the same diameter and same mass, but one is solid and the other is a hollow shell. Describe in detail a nondestructive experiment to determine which is solid and which is hollow.

[10 marks]

Question 4

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(a) A particle moves toward $x = 0$ under the influence of a potential $V(x) = -A|x|^n$, where $A > 0$ and $n > 0$. The particle has barely enough energy to reach $x = 0$. For what values of n will it reach $x = 0$ in a finite time?

[6 marks]

(b) A particle of mass m starts at rest on top of a smooth fixed hemisphere of radius a. Find the force of constraint and the Lagrangian equation of motion

[10 marks]

(c) Consider a region of space divided by a plane. The refractive index in region 1 above the plane is n_1 and in the region n_2 below the plane is n_2. A light ray passes from region 1 to region 2 such that its path in region 1 makes an angle θ_1 with the normal of the plane of separation and an angle θ_2 with the normal. Show that

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

which is the law refraction. In your derivation utilize the Fermat principle which states that a light ray will always travel from one point to another in a medium by a path that requires the least time.

[9 marks]

Question 5

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A double pendulum consists of two simple pendula, one pendulum being suspended from the bob of the other. If the two pendula have equal lengths and have bobs of equal mass and if both pendula are confined to move in the same plane

(a) Find the Lagrangian equation of the system in polar coordinates.

[10 marks]

(b) Two identical harmonic oscillators (with masses M and natural frequencies ω_0) are coupled such that by adding to the system a mass m common to both oscillators the equations of motion become

$$\ddot{x}_1 + (m/M)\ddot{x}_2 + \omega_0^2 x_1 = 0$$

$$\ddot{x}_2 + (m/M)\ddot{x}_1 + \omega_0^2 x_2 = 0$$

Solve this pair of coupled equations, and obtain the frequencies of the normal modes of the system.

[15 marks]