UNIVERSITY OF SWAZILAND

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FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2010/2011

TITLE OF PAPER : COMPUTATIONAL METHODS I

COURSE NUMBER : P262

TIME ALLOWED : THREE HOURS

INSTRUCTIONS :

ANSWER ANY FOUR OUT OF FIVE QUESTIONS. EACH QUESTION CARRIES 25 MARKS.

MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

STUDENTS ARE PERMITTED TO USE MAPLE TO ANSWER THE QUESTIONS.

THIS PAPER HAS SIX PAGES, INCLUDING THIS PAGE.

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Question one

Given the following non-homogeneous ordinary differential equation as

$$\frac{d^2\,y(t)}{d\,t^2}+4\,\frac{d\,y(t)}{d\,t}+15\,y(t)=7\,t^2-3\,t$$

(a) find its particular solution $y_p(t)$, **(8 marks)**

(b) find the general solution $y_h(t)$ for the homogeneous part of the given differential equation, **(4 marks)**

(c) find the general solution $y_g(t)$ for the above given non-homogeneous differential equation, **(2 marks)**

(d) if given initial conditions as $y(0)=5$ *and* $\left.\frac{d\,y(t)}{d\,t}\right|_{t=0}=8$,

find its specific solution of $y(t)$ and plot it for $t=0$ *to* 5 . **(11 marks)**

Question two

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Given the following differential equation as

$$\frac{d^2 y(x)}{dx^2} + 4\frac{d\,y(x)}{dx} + 10\,y(x) = 0$$

utilize the power series method, i.e., setting $y(x) = \sum_{n=0}^{\infty} a_n\, x^{n+s}$ *and* $a_0 \neq 0$,

(a) write down the indicial equations. Find the values of s and a_1 (by setting $a_0 = 1$). **(7 marks)**

(b) Write down the recurrence relation. For all the appropriate values of s and a_1 in (a), set $a_0 = 1$ and use the recurrence relation to find the values of a_n up to the value of a_6 . Thus write down two independent solutions in their polynomial forms. Also write down the general solution of the given differential equation. **(10 marks)**

(c) If the initial conditions are given as $y(0) = -4$ *and* $\left.\frac{d\,y(x)}{dx}\right|_{x=0} = 2$, determine the values of the arbitrary constants of the general solution in (b). Then plot this specific solution for $x = 0$ *to* 1 . **(8 marks)**

Question three

Given the following differential equations for a coupled oscillator system as

$$\begin{cases} \dfrac{d^2 x_1(t)}{dt^2} = -15\, x_1(t) + 6\, x_2(t) \\ \dfrac{d^2 x_2(t)}{dt^2} = 8\, x_1(t) - 28\, x_2(t) \end{cases}$$

(a) set $x_1(t) = X_1\, e^{i\omega t}$ *and* $x_2(t) = X_2\, e^{i\omega t}$, deduce the following matrix equation $A\,X = -\,\omega^2\,X$ where

$$A = \begin{pmatrix} -15 & 6 \\ 8 & -28 \end{pmatrix} \quad and \quad X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} ,$$ **(4 marks)**

(b) (i) find the eigen frequencies of ω , **(4 marks)**

(ii) find the eigen vectors of X , **(4 marks)**

(c) (i) write down the general solutions of $x_1(t)$ *and* $x_2(t)$ in terms of the eigen frequencies and eigen vectors obtained in (b) , **(4 marks)**

(ii) if initial conditions are given as

$$x_1(0) = 5 \ , \quad x_2(0) = 0 \ , \quad \left.\frac{d\,x_1(t)}{dt}\right|_{t=0} = 0 \quad and \quad \left.\frac{d\,x_2(t)}{dt}\right|_{t=0} = 0 \ ,$$

find the specific solutions of $x_1(t)$ *and* $x_2(t)$. Plot both $x_1(t)$ *and* $x_2(t)$ for $t = 0$ *to* 5 and show them in a single display. **(9 marks)**

Question four

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(a) Given the following partial differential equation

$$x^2\, y\, \frac{\partial^2 u(x,y)}{\partial x^2} + x^2\, y\, \frac{\partial u(x,y)}{\partial x} = -\, x\, y^2\, \frac{\partial^2 u(x,y)}{\partial y^2}$$

set $u(x,y) = F(x)\, G(y)$ and utilize the separation of variable scheme to break the above partitial differential equation into two ordinary differential equations.

(8 marks)

(b) The general solution of a one-dimensional wave equation

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u(x,t)}{\partial t^2} = 0 \quad \text{can be written as}$$

$$\begin{aligned} u(x,t) &= \sum_{\forall k} u_k(x,t) \\ &= \sum_{\forall k} \left(A_k \cos(k\,x) + B_k \sin(k\,x)\right)\left(C_k \cos(c\,k\,t) + D_k \sin(c\,k\,t)\right) \end{aligned}$$

where A_k , B_k , C_k & D_k *are arbitrary constants*

(i) by direct substitution, show that the above $u_k(x,t)$ satisfies the given wave equation, **(4 marks)**

(ii) after applying two fixed end conditions and one zero initial speed condition , the above general solution can be deduced to $u(x,t) = \sum_{n=1}^{\infty} E_n \sin\left(\frac{n\pi x}{L}\right) \cos\left(\frac{c\,n\pi t}{L}\right)$ where E_n (n = 1 , 2 , 3 , ...) are arbitrary constants. If c = 2 , L = 6 and the initial position of the string is given as $u(x,0) = \begin{cases} 5x & \text{if } 0 \le x \le 1 \\ -x+6 & \text{if } 1 \le x \le 6 \end{cases}$

find the values of E_1 , E_2 , E_3 , ······ , E_8 . Then plot this specific polynomial solutions of t = 0 , t = 0.1 and t = 0.2 all for the same range of $x = 0$ *to* 6 and show them in a single display.

(13 marks)

Question five

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(a) Given $\frac{d\,y(x)}{d\,x} = \frac{y(x)}{x^2}$ and $y(1) = 5$,

(i) use *dsolve* command to find its specific solution of the above given problem. Then find the value of $y(3)$. **(3 marks)**

(ii) use Euler's method with $h = 0.1$ to find the numerical value of $y(3)$. Compare it with that obtained in (a)(i) and compute their percentage difference. **(5 marks)**

(iii) use Runge-Kutta method with $h = 0.1$ to find the numerical value of $y(3)$. Compare it with that obtained in (a)(i) and compute their percentage difference. **(6 marks)**

(b) Given $\frac{d^2\,y(x)}{d\,x^2} + 4\,\frac{d\,y(x)}{d\,x} + 5\,y(x) = 3\,e^{-x}$ and $y(0) = 2$ & $y'(0) = -1$,

(i) use *dsolve* command to find its specific solution of the above given problem. Then find the value of $y(3)$. **(3 marks)**

(ii) Use Euler's method with $h = 0.3$ to find the numerical value of $y(3)$. Compare it with that obtained in (b)(i) and compute their percentage difference. **(8 marks)**