# UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2009 10

TITLE OF THE PAPER: NUCLEAR PHYSICS

COURSE NUMBER : P442

TIME ALLOWED : THREE HOURS

# **INSTRUCTIONS:**

ANSWER ANY <u>FOUR</u> OUT OF FIVE QUESTIONS.

- EACH QUESTION CARRIES 25 MARKS.
- MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION GIVEN IN THE ATTACHED APPENDIX WHEN NECESSARY.

THIS PAPER HAS **SIX** PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THE PAPER UNTIL THE INVIGILATOR HAS GIVEN PERMISSION.

(i) For  ${}^{16}_{7}N$  (atomic mass=16.0061(u) ) and  ${}^{16}_{8}O$  (atomic mass=15.994915(u)) compute the total binding energy and the binding energy per nucleon. [2]

(ii) Compute neutron and proton separation energies for  ${}^{16}_{7}N$ .

[3]

Given: Atomic mass of  ${}_{6}^{15}C = 15.0105993(u)$ and  ${}_{7}^{15}N = 15.000109(u)$ .

(iii) Define radius of the nuclei. Explain the difference between nuclear matter radius and nuclear charge radius.

[2]

[4]

(iv) For a nucleus of radius R and charge Ze, electrostatic energy is given by  $\frac{3Z(Z-1)e^2}{20\pi\varepsilon_0R}$ 

if we assume the charges to be discrete. Using this for a mirror nuclei, the Coulomb energy difference is given by

$$\left| \Delta E \right| = \frac{6Z \, e^2}{20 \pi \varepsilon_o \, R}$$

where Z refers to the nuclei of lower charge.

Use this formula to estimate electrostatic radius R in the case of  $^{15}_{7}N$  (atomic mass= 15.000109(u))and  ${}^{15}_{8}O$  (atomic mass=15.0105993(u)).

- (B) Describe the principle involved in radioactive dating of rocks and organic specimens. [4]
  - (i) The radioactive decay of  $^{232}_{90}Th$  (with half life 1.4 x  $10^{10}$  yrs )leads eventually [5] to stable  $^{208}_{82}Pb$  . A rock is determined to contain 3.65 gm of  $^{232}_{90}Th$  and 0.75 gm

What is the age of the rock?

(ii) A living organisms has a constant activity due to  ${}^{14}_{6}C$  decay of **0.23 Bq** for [5] each gram of carbon. In 1991 a German tourist found an iceman in the Italian Alps who had become trapped in a glacier. One gram of the material found with the body was found to have an activity of about **0.121 Bq** due to the decay of  ${}^{14}_{6}C$ . What is the age of the iceman?

Q.2.

- **(A)**(i) Define Q value for  $\beta^-$ ,  $\beta^+$  and electron capture. [3]
  - (ii) Can beta decay distinguish matter and anti-matter? Explain your answer. [2]
  - (iii) Explain why two types of selection rules (the Fermi and GT) exist in  $\beta$ -decay. [2]
  - (iv) In  $\beta^+$  decay, positron emission is always accompanied by electron capture process [2] and Auger electrons. Explain.
- (B) Consider the following fusion reaction

$$p + p \rightarrow d + e^+ + v_e$$

Assume the involved protons have negligible kinetic energy.

- Define Q-value in the reaction. (i) [2]
- Ī2Ī (ii) Compute the range of neutrino energies (minimum and maximum).

Note: Mass of proton=938.272 MeV, Mass of deuteron =1875.611 MeV Mass of  $e^+ = 0.511$  MeV. Mass of neutrino  $v_e = 0$  (negligible).

- (C) Classify the following decays according to the degree of forbidden ness
  - (i)  $_{17}^{36}Cl(2^+) \rightarrow _{18}^{36}Ar(3^+)$  [2]
  - (ii)  $_{40}^{97}Zr(1/2^{+}) \rightarrow _{41}^{97}Nb(1/2^{-})$  [2]
- (D) Write brief notes on any **two** of the following: [8]
  - (i) Scintillation detector. Explain why NaI crystal is doped with Tl.(ii) Fission and Fusion.
  - (iii) Cyclotron and Synchrotron.
- **Q.3.** Semi-empirical mass formula for nuclear mass m(Z,A) is given by

$$m(Z,A) = Zm_p + (A-Z)m_n - a_v A + a_s A^{\frac{2}{3}} + a_c Z^2 A^{-\frac{1}{3}} + a_a \frac{(A-2Z)^2}{A} + \delta A^{-\frac{1}{2}}$$

with  $a_v=15.56\;\text{MeV}$  ,  $a_s=17.23\;\text{MeV}$  ,  $a_c=0.697\;\text{MeV}$  ,  $a_a=23.20\;\text{MeV},$  and

- $\delta$  = 11.2 MeV for odd-odd
  - = 0 for odd-even or even-odd.
  - = -11.2 MeV for even -even
- (i) Explain the origin of various terms in the given semi-empirical mass formula. [6]
- (ii) Define the Q-value for symmetric fission of a nuclei of mass m(Z,A). [2] Show that possibility of the spontaneous symmetric fission depends on parameters  $a_s$  and  $a_c$  only. Neglect the contribution from pairing energy term. [8]
- (iii) Define Q value for  $\alpha$ -particle emission in terms of mass m(Z,A) of parent [1] nucleus and m(Z-2,A-4) of daughter nucleus.
  - (a) What is the relation between separation energy for  $\alpha$ -particle and Q-value. [2]
  - (b) What is the relation between Q value and total kinetic energy involved in the  $\alpha$ -decay. [1]
  - (c) Use the semi-empirical mass formula to derive an expression for the Q-value for α-particle emission. Neglect the contribution from pairing energy term. [5]
- Q.4. (A) (i) What do you understand by the term "Magic Numbers"? [2]
- (ii) State the basic assumptions made in the single particle shell model. [4]
- (iii) Explain how the spin and parities are determined using the single particle [4] shell model for even-even, odd-even, even-odd and odd-odd nuclei.
- (iv) Determine the spin and parities for the ground state of the following nuclei by shell model considerations.

(a) 
$$_2^3He$$
 ,  $_{10}^{21}Ne$  ,  $_{31}^{69}Ga$  and  $_{83}^{209}Bi$  . [4]

(b) 
$$_{19}^{38}K$$
 ,  $_{31}^{66}Ga$  , and  $_{37}^{82}Rb$  . [6]

Note: Use the shell model level scheme given in the Appendix.

(B) Describe the significant processes through which the  $\gamma$ -rays in the energy range less than 1.0 MeV primarily interact with matter and its implication on the absorption of radiation in materials.

**Q.5.** (A)  $^{241}_{95}Am$  (5/2+) decays to series of states of  $^{237}_{93}Np$  by a-decay. The ground state of  $^{237}_{93}Np$  is (5/2 +) and following excited states of  $^{237}_{93}Np$  are populated :

33.2(7/2+), 59.527(5/2-), 103(7/2-) and 158.6 (9/2-) keV.

(i) Construct the decay scheme.

[4]

(ii) For each state find the permitted values of  $\ \ell_{a}$  .

[5]

(iii) List all the possible multi-polarities of  $\gamma$ -transitions, indicating in each case most likely transition.

[10]

Note: Do not consider E3, M3 and higher transitions.

### (B) Write short notes on:

[6]

- (a) Non-conservation of parity.
- (b) Internal Conversion

#### @@@END OF EXAMINATION@@@

#### Appendix

# **Selection Rules:**

## (A) $\beta$ -decay:

Type of Transition		ΔΙ	Parity Change
Allowed	Fermi	0	No
	GT	±1 or 0	No
		$(\text{except } 0 \rightarrow 0)$	
1 <sup>st</sup>	Fermi	± 1,0	Yes
Forbidden		( except 0 → 0 )	
	GT	±2, ± 1, or 0	Yes
		( except $0 \rightarrow 0$ ; $1/2 \rightarrow 1/2$ ; $0 \rightarrow 1$ )	
2 <sup>nd</sup>	Fermi	± 2	No
Forbidden	GT	± 3	No

## (B) $\gamma$ - decay:

	E1	E2	E3	E4
Δπ	Yes	No	Yes	No
∆J   ≤	1	2	3	4
	M1	M2	М3	M4
Δπ	No	Yes	No	Yes
Δ] ≤	1	2	3	4

### (C) Useful Information

## PHYSICAL CONSTANTS AND DERIVED QUANTITIES

Speed of light c =  $2.99792458 \times 10^8$  m s<sup>-1</sup> ~  $3.00 \times 10^{23}$  fm s<sup>-1</sup> Avogadro's number  $N_A = 6.02214199(47) \times 10^{26}$  molecules per kg-mole Planck's constant  $h = 6.626068 \ 76(52) \times 10^{-34}$  J s

$$\hbar = 1.054571 \ 596(82) \ x \ 10^{-34} \ J \ s = 0.65821 \ X \ 10^{-21} \ MeV \ s$$
 $\hbar^2 = 41.802 \ u \ MeV \ fm^2$ 
 $\hbar \ c = 197.327 \ MeV \ fm$ 

Elementary charge  $e = 1.602176462(63) \times 10^{-19} \text{C}$  $e^2/4\pi\varepsilon_0 = 1.4400$ MeV fm

 $\frac{e^2}{4\pi\varepsilon_0\hbar c} = 1 / 137.036$ Fine structure constant  $\alpha =$ 

Boltzmann constant  $k = 1.3806503(24) \times 10^{-23} \text{ JK}^{-1} = 0.8617 \times 10^{-4} \text{ eV K}^{-1}$ 

Curie (1 Ci =  $3.7 \times 10^{10}$  dis/sec), is based upon the activity of one gram of radium. Becquerel (Bq = 1 dis/sec)

#### **USEFUL FORMULAE**

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \tau \ln 2$$
 where  $t_{\frac{1}{2}} = \text{half life}$ ,  $\lambda = \text{decay constant and } \tau = \text{mean life}$ .

Energy width of a state of lifetime  $\tau$ :

$$\Gamma$$
= 6.58212 x 10<sup>-22</sup> /  $\tau$ (s) MeV

Non-relativistic speed of mass m with energy E:

$$v = 1.389 \times 10^7 [(E(MeV) / m(u))]^{1/2} \text{ ms}^{-1}$$

Non-relativistic wave number of mass m with energy E:

$$k=2\pi/\lambda=0.21874 [m(u) \times E(MeV)]^{1/2} \text{ fm}^{-1}$$

Wave number for a photon of energy E:

$$k = 2\pi/\lambda = E / \hbar c = E (\text{MeV}) / 197.327 \text{ fm}^{-1}$$

#### MASSES AND ENERGIES

 $m_e$ 

Atomic mass unit  $m_u$  or  $u = 1.66053873(13) \times 10^{-27} \text{ kg}$ 

$$m_u c^2 = 931.494 \text{ MeV}$$

 $= 9.10938188(72) \times 10^{-31} \text{ kg}$ Electron

> $= 5.486 \times 10^{-4} = 1/1823$  $m_e/m_u$  $m_e c^2$ = 0.510998902(21) MeV

 $= 1.67262158(13) \times 10^{-27} \text{ kg}$ Proton  $m_{p}$ 

> = 1.00727647 $m_p / m_u$  $m_p c^2$ = 938.272 MeV

 $= 1.673533 \times 10^{-27} \text{ kg}$ Hydrogen atom

> = 1.007825 $m_{\rm H}/m_u$  $m_H c^2$ = 938.783 MeV

 $= 1.67492716(13) \times 10^{-27} \text{ kg}$ Neutron

> $m_n/m_u$ = 1.00866491578(55) $m_n c^2$ = 939.565 MeV

 $= 6.644656 \times 10^{-27} \text{ kg}$ Alpha particle  $m_{\alpha}$ 

> = 4.001506175  $m_{\alpha}/m_{u}$  $m_{\alpha} c^{2}$ = 3727.379 MeV

### **CONVERSION FACTORS**

Fermi 
$$1 \text{fm} = 10^{-15} \text{ m}$$
  
1 eV=1.6022 x 10<sup>-9</sup> J

Million electron volts 
$$1 \text{ MeV} = 1.602176 \times 10^{-13} \text{ J}$$

$$1 \text{ MeV/c}^2 = 1.783 \times 10^{-30} \text{ kg}$$

Cross section (barn) 
$$1 \text{ b} = 10^{-28} \text{ m}^2$$

Year 1 y = 
$$3.1536 \times 10^7 \text{ s}$$

#### (D) Single particle shell model Level Scheme:

Following diagram gives the energy levels calculated using a realistic potential with spin-orbit interaction according to single particle shell model:

