UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2009/2010

TITLE OF THE PAPER: COMPUTATIONAL METHODS-I

COURSE NUMBER: P262

TIME ALLOWED:
SECTION A: ONE HOUR
SECTION B: TWO HOURS

INSTRUCTIONS:
THE ARE TWO SECTIONS IN THIS PAPER:

- **SECTION A:** IS A WRITTEN PART. ANSWER THIS SECTION ON THE ANSWER BOOK. IT CARRIES A TOTAL OF $\underline{30MARKS}$.
- **SECTION B:** IS A PRACTICAL PART WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. IT CARRIES A TOTAL OF $\underline{70MARKS}$.

Answer *any two questions* from Section A and
***all the questions* from Section B.**
Marks for different sections of each Question are shown in the right hand margin.

THE PAPER HAS 8 PAGES, INCLUDING THIS PAGE.

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Section A – Written Part

Answer *any two questions* in this section

Question 1

(a) With reference to Maple what is the difference between $\{a, b, c, d\}$ and $[a, b, d, c]$?

[2 marks]

(b) Explain the difference between *readdata* and *importdata* command. Given that a data file '*c:/project.txt*' contains two columns of data as given in the table below, read this file in Maple using *readdata* and *importdata*. Give the output for each statement.

x	y
1.0	0.3
2.0	0.57
3.0	0.85
4.0	1.2

[4 marks]

(c) Nine gas particles have speeds of 5.00, 8.00, 12.0, 12.0, 12.0, 14.0, 14.0, 17.0, and 20.8 m/s.

i. Enter the above given data as a *list* in the Maple worksheet, give the list a convenient label.

[2 marks]

ii. Using the *sum* command in Maple to find the average speed of the particles $\overline{v}$ and calculate the root mean square speed v_{rms} of the particles.

[4 marks]

iii. Using the *sequence* statement generate a list of coordinates $[i, v[i]]$, where $v[i]$ is the speed of the i^{th} particle. Plot this list of data points.

[3 marks]

Question 2

(a) Translate the following expressing into Maple

(i) $\sqrt{5}$ upto 10 decimal points

(ii) $1 + 4 + 9 + 16 + 25 + \ldots\ldots\ldots + 10\,000$

(iii) $x^n y^{2/5} e^{-x^2}$

(iv) $\ln(x) + y^b/15!$

[4 marks]

(b) One of the widely used approximation in Physics is the Stirling's formula.

$$\ln n! \approx n \ln n - n$$

Plot the two functions $\ln n!$ and $n \ln n - n$ for n from 0 to 50 on one graph to show that this a good approximation

[3 marks]

(c) The Guassian integral is given by

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx,$$

where $\alpha > 0$.

i. Write Maple commands to evaluate the integral.

ii. To visualize the behaviour of the Guassion distribution plot $e^{-\alpha x^2}$ over an appropriate range of x. Note that since α is an unknown constant you may need to use scaled units.

[4 marks]

(d) Another useful function in physics is the Riemann zeta function $\zeta(s)$ which is usually defined as

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

Plot $\zeta(s)$ such that some details can be seen in the graph for s from 1 to 7, given that it is a divergent series for $s = 1$ and for $s > 3$, $\zeta(s) \approx 1.0$

[4 marks]

Question 3

(a) Explain the difference between

i. > *a: = x+3:* and > *a: = x+3;*

[1 marks]

ii. *sqrt(4.)* and *sqrt(4)*

[1 marks]

iii. *dsolve* and *fsolve*

[2 marks]

iv. *odeplot* and *implicitplot*

[2 marks]

(b) When using Maple to find the roots of the equation

$$x^5 + x^4 - x = 0$$

you obtain the following response
$> a := solve(x\hat{}4 + x\hat{}3 - x, x);$

$$a := 0, RootOf(\_Z^4 + \_Z^2 - 1, index = 1), RootOf(\_Z^4 + \_Z^2 - 1, index = 2),$$
$$RootOf(\_Z + \_Z - 1, index = 3), RootOf(\_Z + \_Z - 1, index = 4)$$

i. What is the meaning of the above response?
ii. How do you extract all the roots?
iii. If we use $fsolve$, Maple displays only the real roots, how can you determine the other complex roots?

[4 marks]

(c) The torsion of a bar is described by the differential equation

$$\frac{d^4\theta}{dx^4} + \tau\theta = 0.$$

Decompose the above equation into a system of four first order equations. Solve the system of equations analytical in Maple given that

$$\theta(x=0) = 0, \ \frac{d\theta(x=0)}{dx} = 1.0, \ \frac{d^2\theta(x=0)}{dx^2} = 0.1, \ \frac{d^3\theta(x=0)}{dx^3} = 1.0.$$

[5 marks]

Section B – Practical Part

Answer ***all the questions*** **in this section**

Question 4

(a) The electrostatic potential $V(x)$ inside a cylinder was measured and the following results were obtained :

$$\begin{aligned} V(0.0) &= 52.640,\ V(0.2) = 48.292,\ V(0.4) = 38.270, \\ V(0.6) &= 25.844,\ V(0.8) = 12.648,\ and\, V(1.0) = 0.0, \end{aligned} \quad (1)$$

where x is the distance from the center of the cylinder in *meters* and V is given in *Volts*.

(i) Sketch a graph of the electric potential $V(x)$ against x.

[5 marks]

Symmetry consideration requires that $V(x)$ be an even function of x. Given that

$$V(x) = \sum_{n=0}^{5} a_{2n}x^{2n} = a_0 + a_2x^2 + a_4x^4 + a_6x^6 + a_8x^8 + a_{10}x^{10},$$

we can evaluate the coefficients a_{2n} from the information given in Eq. (1). It is useful to rearrange the above information into a matrix form **Ab=c** , where

$$\mathbf{A} = \begin{bmatrix} x_0^0 & x_0^2 & x_0^4 & x_0^6 & x_0^8 & x_0^{10} \\ x_1^0 & x_1^2 & x_1^4 & x_1^6 & x_1^8 & x_1^{10} \\ x_2^0 & x_2^2 & x_2^4 & x_2^6 & x_2^8 & x_2^{10} \\ x_3^0 & x_3^2 & x_3^4 & x_3^6 & x_3^8 & x_3^{10} \\ x_4^0 & x_4^2 & x_4^4 & x_4^6 & x_4^8 & x_4^{10} \\ x_5^0 & x_5^2 & x_5^4 & x_5^6 & x_5^8 & x_5^{10} \end{bmatrix}, \mathbf{b} = \begin{bmatrix} a_0 \\ a_2 \\ a_4 \\ a_6 \\ a_8 \\ a_{10} \end{bmatrix}, \mathbf{c} = \begin{bmatrix} V(x_0) \\ V(x_1) \\ V(x_2) \\ V(x_3) \\ V(x_4) \\ V(x_5) \end{bmatrix}$$

(ii) Generate six sequences corresponding to the six rows of the matrix **A**, using that $x_0 = 0.0\,m, x_1 = 0.2\,m, x_2 = 0.4\,m, x_3 = 0.6\,m, x_4 = 0.8\,m, x_5 = 1.0\,m.$

[6 marks]

(iii) Use the linear algebra package in Maple to solve **Ab=c** and thus determine the coefficients a_{2n}.

[7 marks]

(iv) At what distance between $x = 0.0$ and $x = 1.0\,m$ is the electric potential equals to 20 Volts? *You may need to find the roots of the equation*

$$V(x) - 20 = 0.$$

[5 marks]

(b) A typical problem in physics is that one usually can calculate physical quantities for a series of '*small*' systems, but ultimate, one would like to know the results for the *very large* system. In a series of experiments a physicist observes that the energy of finite system E_v is dependent on its size measure v

$$E_2 = -2.000, E_4 = -1.5000, E_6 = -1.4343, E_8 = 1.4128,$$

$$\text{and} E_{10} = -1.4031$$

Now assume that the ground state energy of a finite system is given by

$$E_v = E_\infty + \frac{b_1}{v} + \frac{b_2}{v^2} + ... + \frac{b_r}{v^r} + ...$$

where b_r for $r = 1, 2, 3, 4, ...$ are the model coefficients.

i. Truncate the above series at $r = 4$ determine the energy of the infinite E_∞ system. Express the value of E_∞ in four decimal points. *Note that you may organize the given information in a matrix format justlike in part (a)*

[12 marks]

Question 5

The intensity of light varies as we move away from a light source pass a sharp edge according to the diffraction experiment. The variation of the intensity is given by the expression

$$I(k) = 0.5I_o\{[m(k) + 0.5]^2 + [n(k) + 0.5]^2\}$$

where I_o is the intensity of the incident light. k is proportional to the distance moved from the sharp edge.

$$m(k) = \int_0^k c(w)dw$$

and

$$n(k) = \int_0^k s(w)dw,$$

where $c(w) = \cos(\pi w^2/2)$ and $s(w) = \sin(\pi w^2/2)$ Plot the ratio I/I_o for k from 0 to 5. Comment on the nature of the plot . **Note**: Here $m(k)$ and $n(k)$ are Fresnel integrals and are evaluated numerically and in maple we use the syntax $evalf(int(c(w), w = 0..k))$ and similarly for $n(k)$.

[10 marks]

Question 6

Consider the projectile motion of an unpropelled missile. When air resistance is neglected, the equation of motion of the ball is given by

$$x''(t) = 0, \; y''(t) = -g,$$

where $x(t)$ and $y(t)$ are the horizontal and vertical position of the projectile and g is the acceleration due to gravity. Given that $g = 9.8m/s^2$ and that the missile is launched at $x(t=0) = y(t=0) = 0$, at an angle$\theta = 30°$ with respect to the x-axis, and that the initial speed $v(t=0) = 1000m/s$

(a) Calculate the time dependent position of the missile after launch $x(t)$ and $y(t)$.

[10 marks]

(b) The missile will hit the ground at time $t_f = 2y'(0)/g$ after launched. Plot its trajectory, i.e $x(t)$ versus $y(t)$ for the time $t = 0$ to $t = t_f$.

[5 marks]

(c) In the above analysis we have assumed that the acceleration due to gravity g, is a constant. It is of course dependent on the altitude. Now assume g is a function of the vertical displacement, $g(y) = g_o[1 - \lambda y(t)]$, where $g_o = 9.8m/s^2$ and $\lambda = 7.3 \times 10^{-6} m^{-1}$. Repeat the above calculations adding the new assumptions.

[5 marks]

(d) On one graph plot the trajectory of the missile when g is a constant and when g is a function of the altitude for the time $t = 0$ to $t = t_f$. Compare the two results.

[5 marks]