UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS.

SUPPLEMENTARY EXAMINATION 2008/2009.

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: P461

TIME ALLOWED : THREE HOURS

ANSWER ANY FOUR QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

THIS PAPER CONTAINS FIVE QUESTIONS

QUESTION ONE

(a) (i) A system has three distinguishable particles to be distributed among two energy levels. What are the possible macrostates? (2 marks)

(ii) Use appropriate equations to find the microstates corresponding to each of the above macrostates if:

1. the energy levels are non-degenerate (6 marks)

2. the energy levels have degeneracy two. (6 marks)

(b) Given that the density of states, $g(\varepsilon)d\varepsilon = \frac{2\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$, show that the Maxwell-Boltzmann distribution function in terms of velocity can be written in the differential form

$$n(v)dv = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/(2kT)}v^2 dv$$ (11 marks)

$$Given: \quad e^{\alpha} = \frac{Nh^3}{(2\pi mkT)^{3/2}V}$$

QUESTION TWO.

(a) Show that for a classical system in thermal equilibrium, the Maxwell Boltzmann distribution function $n_s = g_s \exp(\alpha + \beta\varepsilon_s)$, where the symbols have their usual meanings. (12 marks)

(b) (i) A classical non-degenerate system has 15000 particles arranged in two energy levels having energies 1 unit and 2 units. The total energy of the system is 20,000 units. Find the values of α and β for the system. (7 marks)

(ii) Use the values of α and β to find the occupation number of the energy levels. Show that your results agree with the total energy and total number of particles of the system. (6 marks)

QUESTION THREE.

(a) Show that the entropy of a classical perfect gas $S = NkT \ln Z + \frac{E}{T}$ where Z is the partition function.

$$Given: \; Z = \frac{V}{h^3}(2\pi mkT)^{3/2}$$

(6 marks)

(b) Two equal volumes of the same gas having entropy S each, and at the same temperature and pressure are mixed together.
Compute the entropy of the mixture in terms of S using its expression in (a) above.
Do you see any anomaly in your result? If so, by deriving appropriate expressions explain how the anomaly could be resolved.

(14 marks)

(c) Calculate the entropy of one mole of helium gas (^4He) at 300 K from the following data:

Molar volume of the gas	=	22.4×10^{-3} m^3
Avogadro number N	=	6.02×10^{23} mol^{-1}
Plank's constant h	=	6.63×10^{-34} J.S
Mass of He molecule	=	6.65×10^{-27} kg

(5 marks)

QUESTION FOUR.

(a) (i) Set up the partition function of a harmonic oscillator and obtain its mean energy. (6 marks)

(ii) State the basic assumptions Einstein made in developing his theory of specific heat of solids. Hence obtain an expression for the specific heat of an insulator. (9 marks)

$$\textit{Given:}\quad \bar{\varepsilon} = kT^2 \frac{\partial \ln Z}{\partial T}$$

(b) (i) Obtaining appropriate equations to justify your argument discuss how far the above results agree with experiment (7 marks)

(ii) Draw a schematic diagram to show how the specific varies with temperature according to Einstein's theory. (3 marks)

QUESTION FIVE.

(a) (i) Define Fermi energy. (2 marks)

(ii) "At ordinary temperatures metals are degenerate". Explain. (4 marks)

(b) (i) Given that the density of states for fermions is:

$$g(\varepsilon)d\varepsilon = \frac{4\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$$

where symbols have their usual meanings, show that the Fermi energy of a system of fermions:

$$\varepsilon_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$$ (8 marks)

(ii) Calculate the Fermi energy of a metal having density 8.5×10^2 kg m^{-3} and atomic weight 40.

(5 marks)

(c) The electronic contribution to specific heat capacity is given as $C_V = 3Nk\,T / T_F$

Define T_F in this equation. Calculate its value at 300 K for a metal with Fermi energy 3.12 eV. Comment on its effect on the specific heat. (6 marks)

Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Plank's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan constant	σ	5.67×10^{-8} Wm^{-2}K^{-4}
Atmospheric pressure		1.01×10^{5} Nm^{-2}
Mass of ${}_2^4$He atom		6.65×10^{-27} kg
Mass of ${}_2^3$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 l mol^{-1}

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}.$$