# UNIVERSITY OF SWAZILAND

## FACULTY OF SCIENCE

## DEPARTMENT OF PHYSICS

## MAIN EXAMINATION 2008-09

**TITLE OF THE PAPER:** **COMPUTATIONAL METHODS-I**

**COURSE NUMBER : P262**

**TIME ALLOWED : SECTION A: ONE HOUR.**
**SECTION B: TWO HOURS.**

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***INSTRUCTIONS:***

**THERE ARE TWO SECTIONS IN THIS PAPER:**

***SECTION A*** **: THIS IS WRITTEN PART ON YOUR ANSWER BOOK. CARRIES A TOTAL OF <u>30</u> MARKS.**

***SECTION B*****: THIS IS A PRACTICAL PART WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. CARRIES A TOTAL OF <u>70</u> MARKS.**

ANSWER **ANY TWO** QUESTIONS FROM **SECTION A** AND

**ALL** THE QUESTIONS FROM **SECTION B.**

MARKS FOR EACH QUESTION ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS **<u>FOUR</u>** PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THE PAPER UNTIL THE INVIGILATOR HAS GIVEN PERMISSION.

## SECTION A
## (Written Section)

**Q.1.**

**(A)** Explain the difference between [5]

(i) ***y1:= a*x +b*** and ***y2:= (a,b,x) → a*x+b*** .

(ii) ***diff*** and ***Diff*** .

(iii) ***solve*** and ***fsolve*** .

(iv) ***Pi*** and ***pi*** .

(v) the symbol **( )** and **[ ]** .

**(B)** Use Maple statements to find the roots of the equation

**sin(x))^2-x^3=0**

gives the following response [3]

```
> s:=solve((sin(x))^2-x^3=0,x);
```

$$s := \mathrm{RootOf}(-\sin(\_Z)^2 + \_Z^3)$$

i. What is the meaning of the above response?
ii. How do you extract the root?
iii. How do you determine the number of real roots in such a situation?

**(C)** Write Maple statements to calculate [2]

(i) $\int_0^2 x^3 dx$

(ii) $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}+\frac{1}{7^2}$

**(D)** The steady state temperature distribution T(x) along a particular thin bar of unit length is given by

$$\frac{d^2T}{dx^2}=10(T-20)$$

The initial conditions are at x=0, T=$100^0$ C and $\frac{dT}{dx}=1.0$.

Using the Maple commands:

(i) Find the exact solution for T(x). Plot the solution for the interval $0 \le x \le 1$. [2]

(ii) Solve the equation numerically using default numerical method available in Maple. Plot the solution in the interval $0 \le x \le 1$. [2]

(iii) Plot solutions of (i) and (ii) on one graph. [1]

**Q.2.** Van der Waal's equation for an **imperfect** gas leads to following equation for volume V for a given pressure P and temperature T

$$V^3-\left(b+\frac{RT}{P}\right)V^2-\frac{a}{P}V-\frac{ab}{P}=0$$

R=8.3149 x $10^3$ J $kg^{-1}$ $mole^{-1}$ $K^{-1}$ .

Constants a and b are related to critical pressure $P_c$ and temperature $T_c$ . They are given by the relation:

$$a = \frac{27R^2T_c^2}{64P_c} \quad , \quad b = \frac{RT_c}{8P_c}$$

For $CO_2$ , $T_c$ = 304.26 K and $P_c$ = 7.40 x $10^6$ Pa.
Write Maple code to

(a) Calculate a and b. [2]

(b) Use the calculated values of a and b to calculate volume V using **fsolve** command for the imperfect gas at temperature $300^0$ K and at pressures 5 x $10^6$ Pa to 10 x $10^6$ Pa with an increment of 1 x $10^6$ Pa. [8]
**Note:** *You need to use for loop.*

(c) For each value of V and P calculate the speed of sound ***u*** in an imperfect gas given by the expression [5]

$$u = \left\{ \frac{C_p}{\mu C_v}\left[\frac{RT}{(v-b)^2} - \frac{2a}{v^3}\right]\right\}^{1/2}$$

where for $CO_2$ , μ=molecular weight 44.01 and $C_p / C_v$ = 1.3.

***Note: You may combine (b) and (c) in one group if necessary.***

**Q.3. (a)** Use Maple statements to generate 10 data points *( $x_i$ , $y_i$ )* using the equation [5]

$$y_i = 0.1 + 9.8\, x_i$$

for $x_i = x_{i-1} + \Delta x$ for $i = 0\ldots10$ with $x_0=1.0$ and $\Delta x=0.1$ .

(**b**)Using the above data points, write a program in Maple to calculate [10]

$$\sum_{i=0}^{10} x_i \ , \sum_{i=0}^{10} y_i \ , \sum_{i=0}^{10} x_i^2 \ , \sum_{i=0}^{10} y_i^2 \ , \sum_{i=0}^{10} x_i y_i \ , \text{and} \sum_{i=0}^{10} (x_i^2 - x_i y_i)$$

**Note:** *You have to use for loop.*

## SECTION B
## (Practical Section)

**Q.4.**
**(A)** $I(\lambda)$ , the intensity distribution of radiation from a blackbody at a temperature T , may be represented by a formula

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^3}\frac{1}{\exp(hc/\lambda kT)-1}$$

where $\lambda$ wavelength in metres, T is the temperature in Kelvin, c= 3 x $10^8$ $ms^{-1}$, h= 6.63 x $10^{-34}$ Js and k= 1.38 x $10^{-23}$ $JK^{-1}$.

Plot following on one graph $I(\lambda)$ at temperature T=2000 K over

i. the visible wavelength range 0.4 μm to 0.7μm in blue and [10]

ii. wavelength range 0.3 μm to 10 μm jn any color. [2]

**Note:** 1 μm = $10^{-6}$ m

**(B)** Intensity of the light varies as we move away from the edge of a sharp edge and results into a diffraction pattern. The variation of the intensity is given by the expression

$$I = 0.5\, I_0 \left[ (c(v)+0.5)^2 + (s(v)+0.5)^2 \right]$$

where $I_0$ = intensity of the incident light,

$v$ = proportional to the distance moved from the sharp edge.

$$c(v) = \int_0^v \cos(\pi w^2/2)\, dw \quad \text{and} \quad s(v) = \int_0^v \sin(\pi w^2/2)\, dw$$

Plot the ratio $I/I_0$ for $v$= 0 to 5. [10]
Comment on the nature of plot. [3]

**Note:** Here c(v) and s(v) are Fresnel integrals and are evaluated numerically and in Maple we will use the syntax

***evalf(int(cos(Pi*w^2/2),w=0..v))***

and similarly for s(v).

**(C)** In an R,L,C circuit, frequency f is given by the equation [10]

$$f^2 = \frac{1}{LC} - \frac{R^2}{4C^2}$$

For *L= 2.0 E-14 H*, *R=4E+3 Ω*, use Maple command fsolve and for loop to calculate *C* for frequencies 100 Hz to 500 Hz in steps of 50 Hz.

**Q.5.** An object of mass ***m*** rests on an idealized frictionless surface and is constrained by a spring with a spring constant ***k***. The motion of the mass is damped by the force of air resistance, taken to be proportional to the speed of the object. The acceleration of the object is then given by

$$a(t) = -\frac{1}{m}(kx(t) + cv(t))$$

where ***a(t)*** , ***v(t)*** and ***x(t)*** are the acceleration, velocity and position of the object respectively at time ***t***. ***c*** is a constant related to air resistance. Take ***k***=1 $Nm^{-1}$, ***m***=1kg and ***c***=0.2 $kgs^{-1}$.

Note: $a(t) = \frac{dv(t)}{dt}$ *and* $v(t) = \frac{dx(t)}{dt}$

Given the initial condition at ***t** = 0 , **x**= 1m and **v** = 0*, solve the differential equations for ***x(t)*** and ***v(t)*** for the interval $0 \le t \le 1$ **s** with Maple commands for solving differential equation

(a) To find exact (analytic) solution. Plot the solutions for x(t) and v(t) on one graph for the interval $0 \le t \le 1$ **s**. [15]

(b) To find numerical solution for x(t) and v(t). For this solution convert the second order differential equation for x(t) into two first order differential equations involving x(t) and v(t). [15]
Plot both solutions on one graph for the interval $0 \le t \le 1$ **s**.

(c) Plot v(t) vs x(t) for the interval $0 \le t \le 10$ **s**. [5]

**@@@@END OF EXAMINATION@@@@**