UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION	**:**	**MAY 2009**
TITLE OF PAPER	**:**	**THERMODYNAMICS**
COURSE NUMBER	**:**	**P242**
TIME ALLOWED	**:**	**THREE HOURS**
INSTRUCTIONS	**:**	**ANSWER ANY FOUR OUT OF FIVE QUESTIONS**
		EACH QUESTION CARRIES 25 MARKS
		MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

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INFORMATION

Universal gas constant	$= 8.31\ J\ mol^{-1}K^{-1}$
Specific heat of water	$= 4190\ J\ kg^{-1}K^{-1}$
Density of water	$= 10^3\ kgm^{-3}$
Specific heat of iron	$= 448\ J\ kg^{-1}K^{-1}$
Avogadro's number	$= 6.02 \times 10^{23}$ molecules.mol^{-1}
Boltzmann constant	$= 1.38 \times 10^{-23}\ JK^{-1}$
Stefan-Boltzmann constant	$= 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$
1 atmosphere	$= 1.013 \times 10^5 Nm^{-2}$
Thermal conductivity of air	$= 0.0234\ Wm^{-1}K^{-1}$

QUESTION 1

(a) A vessel in the shape of a spherical shell has an inner radius a and outer radius b. The wall has a thermal conductivity k. If the inside is maintained at a temperature T_1 and the outside is at a temperature T_2, show that the rate of heat flow between the surfaces is

$$\frac{dQ}{dt} = \left(\frac{4\pi kab}{b-a}\right)(T_1 - T_2)$$ (10 marks)

(b) The brick wall ($k = 0.80$ W/m.K) of a building has dimensions of 4.0 m x 10.0 m and is 15 cm thick. How much heat (in joules) flows through the wall in a 12-h period when the average temperatures inside and outside are, respectively, 293 K and 278 K? (4 marks)

(c) The solar constant, or the quantity of radiation received by the earth from the sun is 1.4 kW.m^{-2}. Assume that the sun may be regarded as an ideal radiator. The ratio of the radius of the earth's orbit to the radius of the sun is 216.

Calculate the surface temperature of the sun. (7 marks)

(d) A Thermos flask in the shape of a cylinder has an inner radius of 4.0 cm, outer radius of 4.5 cm, and length of 30.0 cm. The insulating walls have a thermal conductivity equal to 8.4×10^{-3} W/m.K. One litre of hot coffee at 363 K is poured into the bottle.

Determine the rate at which heat flows through the walls of the flask if the outside wall remains at 293 K. (4 marks)

QUESTION 2

(a) (i) Use the ideal gas law and the first law of thermodynamics to show that for a reversible adiabatic process

$$pV^{\gamma} = constant$$ (8 marks)

(ii) Use the expression in (a) (i) to demonstrate that

$$T^{\gamma} p^{1-\gamma} = constant$$ (3 marks)

where the symbols have the usual meaning.

(b) One mole of an ideal gas is initially at a pressure of 2.0 atm and a volume of 0.30 L. In its final state the gas is at a pressure of 1.5 atm and a volume of 0.80 L. For the paths *IAF*, *IBF*, and *IF* in Fig. 2.1, calculate the work done by the gas. (8 marks)

(c) A volume of 12 litres of CO_2 at 1 atm pressure and 27 °C is compressed adiabatically (and slowly) to 1 litre.

(i) Find the final pressure and temperature of the gas (4 marks)
(ii) What is the external work done on the gas during the compression. (2 marks)

(Note: $\gamma = 1.33$ for CO_2)

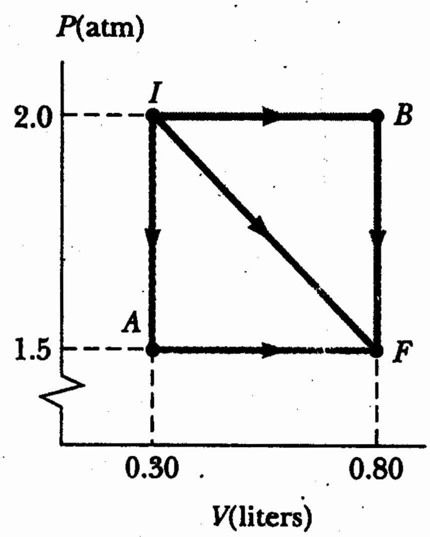

Fig. 2.1

QUESTION 3

(a) Show that the efficiency of a heat engine operating in the Carnot cycle illustrated in Fig. 3.1, using an ideal gas as the medium, is given by

$$\eta = 1 - \frac{T_C}{T_H}$$ (10 marks)

(b) (i) The highest theoretical efficiency of a gasoline engine, based on the Carnot cycle, is 30%. If this engine expels its gases into the atmosphere, which has a temperature of 300 k, what is the temperature in the cylinder immediately after combustion? (2 marks)

(ii) If the heat engine absorbs 837 J of heat from the hot reservoir during each cycle, how much work can it perform in each cycle? (5 marks)

(c) Suppose 1.00 kg of water at 0 °C is mixed with an equal mass of water at 100 °C. After equilibrium is reached, the mixture has a uniform temperature of 50 °C. What is the change in entropy of the system? (8 marks)

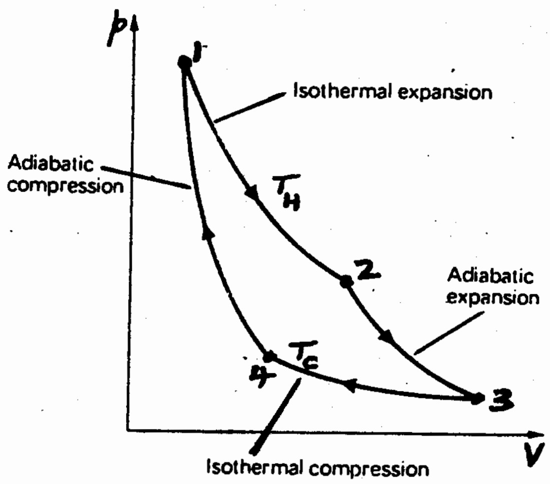

Fig. 3.1

QUESTION 4

(a) One mole of a gas obeying van der Waals' equation of state is compressed isothermally. At some critical temperature, T_{cr}, the isotherm has a point of zero slope, as in Fig. 4.1. That is, at $T = T_{cr}$,

$$\frac{dp}{dv} = 0 \quad and \quad \frac{d^2p}{dv^2} = 0$$

Using the van der Waals' equation of state and these conditions, show that at the critical point, $P_{cr} = a/27b^2$, $v_c = 3b$, and $T_{cr} = 8a/27Rb$. (8 marks)

(b) The van der Waals' equation of state can be written in the form

$$pv = RT\left(1 - \frac{b}{v}\right)^{-1} - \frac{a}{v}$$

With the aid of the binomial theorem, determine the first and second virial coefficients. (6 marks)

(c) The constant b in van der Waals' equation, for a nitrogen gas, is 3.9×10^{-5} /mol. Estimate the diameter of a nitrogen molecule. (4 marks)

(d) Imagine a CO_2 gas in a rigid vessel. The temperature of the gas is 80 °C and its molal specific volume is 0.18 m^3/mol. The van der Waals' constants a and b for CO_2 are 3.6×10^{-1} Nm^4/mol^2 and 4.3×10^{-5} m^3/mol, respectively.

Estimate the pressure exerted by the gas onto the walls of the vessel using

(i) the van der Waal's equation (4 marks)

(ii) the ideal gas law (3 marks)

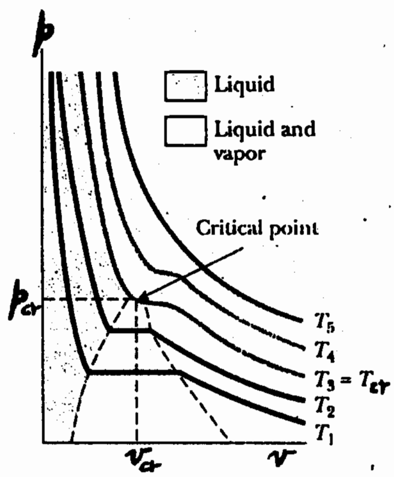

Fig. 4.1

QUESTION 5

(a) (i) State the second law of thermodynamics. (2 marks)

(ii) Explain what this law means, with reference to a thermal power station. (4 marks)

(iii) What are the factors that affect the efficiency of automobile engines? Mention three of these factors. (3 marks)

(b) Consider a Carnot engine that takes 6300 J of heat each cycle from the high-temperature reservoir at 500 K and gives out 3780 J to the low-temperature reservoir.

(i) Calculate the temperature of the low-temperature reservoir. (4 marks)

(ii) What is the thermal efficiency of the engine and (2 marks)

(iii) How much external work is done during each cycle? (2 marks)

(c) The motor in a refrigerator has a power output of 200 watts. The freezing compartment is at -3 °C and the outside air is 27 °C.

Assuming that the refrigerator behaves like a Carnot refrigerator, what is the maximum amount of heat that can be extracted from the freezing compartment in 10 minutes? (8marks)