UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2008/09

TITLE OF PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

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QUESTION 1

(a) (i) Use a clear diagram as an aid to show the polar unit vectors $\hat{r}$ and $\hat{\theta}$ in terms of Cartesian unit vectors $\hat{i}$ and $\hat{j}$. **(6 marks)**

(ii) Show that $\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}$. **(6 marks)**

(iii) Show that $\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{\theta}$. **(6 marks)**

(b) Derive the kinematic equation

$$\upsilon^2 = \upsilon_0 + 2a(x - x_0)$$ **(7 marks)**

QUESTION 2

(a) The Atwood's machine shown in Figure 1. has a pulley of negligible mass (or negligible moment on inertia). Mass M is greater than mass m.

(i) Make the force diagram for each mass and find the tension in the rope in terms of m, M, and g. **(6 marks)**

(iii) What is the acceleration of the mass M? **(3 marks)**

(b) A particle of mass m slides without friction on the inside of a cone. The axis of the cone is vertical, and gravity is directed downward. The apex half angle of the cone is θ as shown in Figure 2. The path of the particle happens to be a circle in a horizontal plane. The speed of the particle is a constant υ_0.

(i) Make a useful force diagram for the body from which you can obtain useful equations of motion.. **(6 marks)**

(ii) Find the radius r of the circular path in terms of υ_0, g and θ. **(10 marks)**

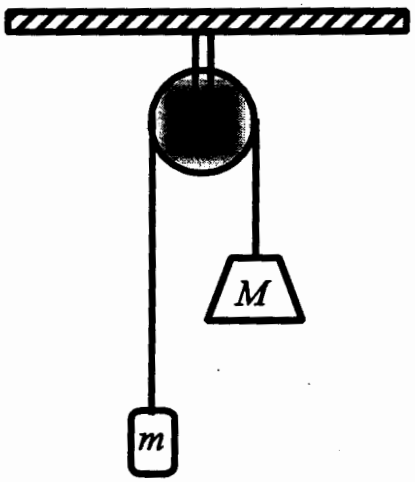

Figure 1.

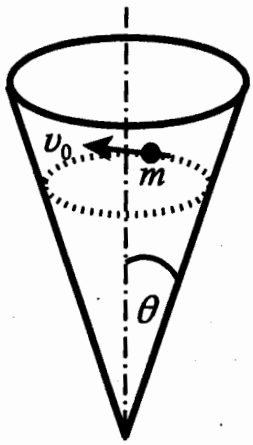

Figure 2.

QUESTION 3

(a) Find the centre of mass of a thin rod of length L where the density varies with distance according to the equation $\lambda = \lambda_0 x^2/L^2$ from one end, where λ_0 is a constant. **(9 marks)**

(b) An empty rail car of mass M_0 starts from rest under an applied force F. At the same time, maize begins to fill the car at a steady rate $dm/dt = b$ from a hopper at rest along the rail tracks. (See Figure 3). Use the mass and momentum transport method to find the velocity when a mass, m, of maize has been transferred to the rail car. **(16 marks)**

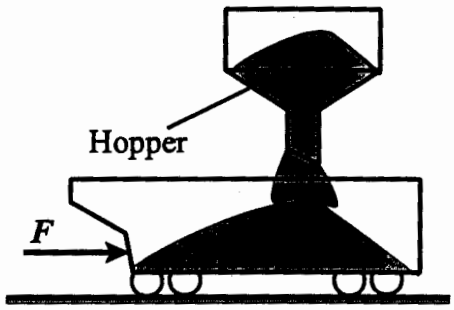

Figure 3.

QUESTION 4

(a) A mass m is projected upward from the earth's surface with an initial velocity υ_0. Use the work energy-theorem to determine the highest point r_{max} reached by the body in terms of the radius of the earth R_E, mass υ_0 and the gravitational acceleration g. Note that the gravitational acceleration varies with altitude and that $g = GM_E/R_E^2$. **(10 marks)**

(b) A particle is to be taken from point (0,0) to (0,1), as shown in Figure 4. by a force

$\vec{F} = A(xy\hat{i} + y^2\hat{j})$, where A is a constant with appropriate unit.

Path 1 is straight from (0,0) to (0,1). Path 2 is from (0,0) to (1,0) to (1,1) and then to (0,1)

(i) What is the work done when the particle is moved along Path 1? **(2 marks)**
(ii) What is the work done when the particle is moved along Path 2? **(6 marks)**
(iii) From your results what can you say about the force? **(2 marks)**

(c) Find the potential energy function for a particle moving along the x-axis under a force given by the equation

$$F = -kx^2 + c,$$

where k and c are constants with appropriate units. **(5 marks)**

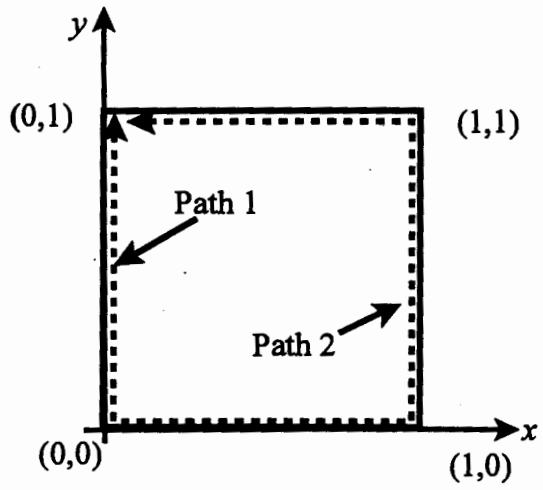

Figure 4.

QUESTION 5

(a) find the moment of inertia of a disc of mass M and radius R about its axis of symmetry. **(10 marks)**

(b) An object moves with angular momentum given by

$$\vec{L} = MRV\sin\alpha(\cos\omega t\hat{i} + \sin\omega t\hat{j}) + MRV\cos\alpha\hat{k},$$

where M, R, V and ω are constants.

(i) Find the torque on the object. **(8 marks)**

(ii) What is the magnitude of the torque? **(2 marks)**

(c) Derive the following equation for angular motion

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2. \quad \textbf{(5 marks)}$$