UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMETARY EXAMINATION 2007/2008

TITLE OF THE PAPER: COMPUTATIONAL PHYSICS-II

COURSE NUMBER : P482

TIME ALLOWED : SECTION A: ONE HOUR.
SECTION B: TWO HOURS.

INSTRUCTIONS:

THERE ARE TWO SECTIONS IN THIS PAPER:

SECTION A **: THIS IS WRITTEN PART ON YOUR ANSWER BOOK. CARRIES A TOTAL OF 30 MARKS.**
SECTION B **: THIS IS A PRACTICAL PART WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. CARRIES A TOTAL OF 70 MARKS.**

ANSWER **ANY TWO** QUESTIONS FROM **SECTION A** AND **ALL** THE QUESTIONS FROM **SECTION B.**

MARKS FOR EACH QUESTION ARE SHOWN IN THE RIGHT-HAND MARGIN.

USE THE INFORMATION GIVEN IN THE ATTACHED **APPENDIX** WHEN NECESSARY.

THIS PAPER HAS FOUR PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THE PAPER UNTIL THE INVIGILATOR HAS GIVEN PERMISSION.

SECTION A
(Written Section)

Instruction: Use the information given in Appendix when necessary.

Q.1. (a) Explain the term random numbers. Why the random numbers generated on a computer are called as psuedo-random numbers. [2]

(b) State advantages and disadvantages of Monte-Carlo integration. [2]

(c) Write a procedure in Maple code to calculate an integral [11]

$$\int_0^1 f(x)dx$$

for any function f(x).

Q.2. Consider random walk problem in two dimensions on a square lattice. The walk starts at the origin which is at a lattice point approximately at the centre of the lattice and assume length of each step to be of one unit . Each point on the lattice is recognized by the coordinates ***(x,y)*** from the origin. [15]

Write a program in pseudo code to determine the distance $d = \sqrt{x^2 + y^2}$ for N =500,1000,2000,4000 where N=number of steps taken for a walk.

Note: *Use* ***for*** *loop to increment value of N from the initial value.*

Q.3. The equation of motion for the driven oscillator with damping force is given by

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + k\,x = F_0\cos(\omega t)$$

where k is the force constant, b is damping constant , and F_0 amplitude of driving force of angular velocity ω .

Assume initial conditions to be at *t=0* , *x(t=0)=0* and $\left.\frac{dx}{dt}\right|_{t=0} = 0$.

(a) Convert the given equation into two first order differential equations. [2]

(b) Write an algorithm in pseudo code to find the solution x(t) on the interval $0 \le t \le 25$ at n=200 points. [10]
Extend the above pseudo code for n to n+10 and verify if the integral is convergent with a precision of 0.01. [3]

SECTION B
(Practical Section)

Instruction: Use the information given in Appendix when necessary.

Q.4. For a charged thin wire along the x-direction which extends from *x= a to b* the potential *V* at any point (x_0, y_0) is given by the integral [35]

$$V = \int_a^b \frac{\sin(\pi k x)}{\left[(x - x_0)^2 + y_0^2\right]^{1/2}} dx$$

Assume that *a= -3.5m* and *b=3.5m* and *k=0.1* .

Write a program to calculate the potential at $x_0=1.2$, $y_0=1.2$ using the **Simpson method**. Include in your program a criterion (precision) to confirm that the integral is convergent , that is, calculate the integral with N steps and then with N+20 steps and find the difference between the integrals and if the difference is small then the integral is convergent.
Consider the precision to be 0.001.
Assume the initial value of N=40.
Note: Use the evaluated value of Pi

Q.5. Consider random walk problem in two dimensions on a square lattice. The walk tarts at the origin which is at a lattice point approximately at the centre of the lattice and assume length of each step to be of one unit . Each point on the lattice is recognized by the coordinates ***(x,y)*** from the origin. At a distance of 25 units, in any direction, there is a point of no return , that is, the walk terminates as soon as one reaches 25 units or more.

(a) Write a program and execute it to determine the number of steps required to reach the point of no return. [25]

Consider initial value of for the number of steps to be N=1000.

(b) Convert the program into a procedure which is valid for any N, any distance of point of no return. Verify its working with N=1000, 2000. [10]

Note: `stats[random,uniform](n):` **`# produces n uniform random numbers in the range (0,1).`**

@@@@END OF EXAMINATION@@@@

Appendix:

1. Solution of First Order Differential Equation with initial Conditions:

The equation is of the form $\frac{dy}{dx} = f(x,y)$ with the given initial boundary condition $y(x_0) = \alpha$.

(i) Euler's Method:

$$y_{i+1} = y_i + h\,f(x_i, y_i) \qquad \text{where } h = x_{i+1} - x_i$$

(ii) Fourth Order RK-Method:

$$y_{i+1} = y_i + (k_1 + 2k_2 + 2k_3 + k_4)/6$$

where

$$k_1 = h\,f(x_i, y_i)$$
$$k_2 = h\,f(x_i + 0.5\,h,\ y_i + 0.5\,k_1)$$
$$k_3 = h\,f(x_i + 0.5\,h,\ y_i + 0.5\,k_2)$$
$$k_4 = h\,f(x_i + h,\ y_i + k_3) \qquad \text{and} \quad h = x_{i+1} - x_i$$

2. Numerical Integration:

(A) Simpson Rule:

$$\int_a^b f(x)dx = \frac{h}{3}[f(a) + f(b) + 4(f_1 + f_3 + f_5 + \ldots\ldots + f_{n-1}) + 2(f_2 + f_4 + f_6 + \ldots + f_{n-2})]$$

where $f_0 = f(a)$, $f_n = f(b)$, $f_1 = f(a+h)$, $f_2 = f(a+2h)$......*etc. and* n=*even integer.*

(B) Monte Carlo Evaluation of Integrals:

Integral of the form $F = \int_a^b f(x)dx$ is given by

$$F = \frac{(b-a)}{n}\sum_{i=1}^{n} f(x_i)$$

where x_i is the i^{th} random number of n random numbers distributed uniformly in the interval (a,b). The standard deviation can be estimated from the points sampled in evaluating the integral by

$$\sigma_n = \left[\frac{\frac{1}{n}\sum_{i=1}^{n}[f(x_i)]^2 - [\frac{1}{n}\sum_{i=1}^{n} f(x_i)]^2}{n-1}\right]^{1/2}$$

with 68.3% confidence.