UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS.

MAIN EXAMINATION 2007/2008

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: P 461

TIME ALLOWED : THREE HOURS

ANSWER ANY **FOUR** QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

THIS PAPER CONTAINS FIVE QUESTIONS

Question One.

(a) (i) What is meant by ***statistical weight*** of a system of particles? (2 marks)

(ii) What is its significance on the properties of the system? (2 marks)

(b) (i) A system has 6 distinguishable particles arranged in 2 non-degenerate energy levels. What are the possible macrostates? (2 marks)

(ii) Find the number of microstates corresponding to each macrostate and hence determine the most probable configuration. (8 marks)

$$W = N!\,\pi_s\left(\frac{g_s^{\,n_s}}{n_s!}\right)$$

(c) (i) Define 'density of states' in phase space. (2 marks)

(ii) Derive an expression for the volume element in phase space in terms of energy and show that density of states

$$g(\varepsilon)d\varepsilon = \frac{2\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon,$$

where symbols have their usual meanings. (9 marks)

Question Two.

(a) The distribution function for system of classical particles in equilibrium at temperature T is given as: $n_s = g_s e^{\alpha + \beta\varepsilon_s}$, where symbols have their usual meanings. Show that the numerical value of the constant β in this equation is equal to - 1/(kT), where k is the Boltzmann constant. (10 marks)

(b) Derive the following expressions for an ideal gas in terms of its partition function Z.

(i) the total energy $E = NkT^2\left(\frac{\partial \ln Z}{\partial T}\right)_V$ (6 marks)

(ii) Pressure $P = NkT\left(\frac{\partial \ln Z}{\partial V}\right)_T$ (3 marks)

(c) Given that the partition function of a classical system $Z = aVT^4$, where a is a constant, find the values of:

(i) total energy

(ii) pressure (6 marks)

Question Three

(a) Show that in a system of bosons, for the most probable configuration, the distribution of the bosons can be represented as:

$$n_S = \frac{g_S}{e^{-(\alpha+\beta\varepsilon_S)} - 1}$$ where the symbols have their usual meanings

(Given that the weight of a system of bosons, $W = \prod_S \frac{(g_S - 1 + n_S)!}{(g_S - 1)!n_S!}$)

(12 marks)

(b) (i) State what each symbol represents in the Bose - Einstein condensation equation:

$$\frac{N'}{N} = \left(\frac{T}{T_B}\right)^{3/2}.$$

(ii) Find the relationship between the number of particles N_0 in the ground state and the temperature.

(iii) Draw a sketch to show how N_0 varies with temperature.

(6 marks)

(c) In a Bose - Einstein condensation experiment, 10^7 rubidium (atomic mass = 85.47 g/mol) atoms were cooled down to a temperature of 200 nK. The atoms were confined to a volume of 10^{-15} m^3 .

(i) Calculate the condensation temperature. (4 marks)

(ii) Find how many atoms were in the ground state at 200 nK.

given $T_B = \frac{h^2}{2\pi mk}\left(\frac{N}{2.612V}\right)^{2/3}$ (3 marks)

Question Four

(a) The quantum statistical expression derived by Max Planck for the spectral distribution of energy from a black body is expressed as:

$$E(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT} - 1}$$

Derive the spectral distribution under short and long wavelength limits.

(8 marks)

(b) (i) Use the above Planck's distribution function to show that the total energy radiated is proportional the fourth power of the absolute temperature of the body. (See appendix for definite integrals) (10 marks)

(ii) Given that the proportionality constant in the above expression for total energy is equal to σ (4/c), where σ is the Stefan-Boltzmann constant, and c the velocity of light, calculate the value of σ.

(7 marks)

Question Five

(a) (i) Given that the density of states for a system of fermions:

$$g(\varepsilon)d\varepsilon = \frac{4\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$$, show that the Fermi energy of the system is:

$$\varepsilon_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$$, where symbols have their usual meanings.

(8 marks)

(ii) Use the above expression for the Fermi energy to show that the average kinetic energy of a particle in a Fermi gas at 0 K is (3/5) times the Fermi energy.

[Hint: average energy = $(1/N)\int \epsilon\, n(\epsilon)d\epsilon$] (6 marks)

(b)

(i) Define the Fermi temperature T_F (2 marks)

(ii) Calculate T_F at 300 K for a metal with Fermi energy 3.12 eV.

(3 marks)

(iii) The electronic contribution to specific heat capacity is given as $C_V = 3NK\,T / T_F$. Comment on the effect of T_F as calculated in (ii) above on the specific heat .

(2 marks)

(c) Calculate the fermi energy of a metal (in electron volts) having an electron density of $5x10^{28}$ m^{-3}. (4 marks)

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Plank's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan constant	σ	5.67×10^{-8} $Wm^{-2}K^{-4}$
Atmospheric pressure		1.01×10^{5} Nm^{-2}
Mass of ${}_2^4$He atom		6.65×10^{-27} kg
Mass of ${}_2^3$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 l mol^{-1}