UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2006/2007

TITLE OF THE PAPER: COMPUTATIONAL PHYSICS-II

COURSE NUMBER : P482

TIME ALLOWED : *SECTION A: ONE HOUR.*
SECTION B: TWO HOURS.

INSTRUCTIONS:

THERE ARE TWO SECTIONS IN THIS PAPER:

SECTION A **: THIS IS A WRITTEN PART ON YOUR ANSWER BOOK. CARRIES A TOTAL OF <u>30</u> MARKS.**
SECTION B**: THIS IS A PRACTICAL PART WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. CARRIES A TOTAL OF <u>70</u> MARKS.**

ANSWER **ANY TWO** QUESTIONS FROM **SECTION A** AND **BOTH** THE QUESTIONS FROM **SECTION B.**

MARKS FOR EACH QUESTION ARE SHOWN IN THE RIGHT-HAND MARGIN.

USE THE INFORMATION GIVEN IN THE ATTACHED APPENDIX WHEN NECESSARY.

THIS PAPER HAS <u>FOUR</u> PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THE PAPER UNTIL THE INVIGILATOR HAS GIVEN PERMISSION.

SECTION A
(Written Section)

Q.1. (a) A multiple integral of the form $I = \iint_R f(x,y)\,\rho(x,y)dxdy$ where the region $R = \{(x,y) | a \le x \le b, c \le y \le d\}$, can be calculated by use of Monte Carlo method and the algorithm for the estimation of the integral is

$$I = \frac{(b-a)(d-c)}{n}\sum_{i=1}^{n} f(x_i,y_i)\,\rho(x_i,y_i)$$

Here n random numbers for x_i *and* y_i are generated ***independently*** in the interval $a \le x \le b$ *and* $c \le y \le d$.

(i) Explain what you understand by random numbers? [2]

(ii) Write a psuedo code for calculating I. [5]

(b) A plane lamina is defined to be a thin sheet of continuously distributed mass. If $\rho(x,y)$ is the function describing the density of a lamina having a shape of a region R in the xy-plane, then the center of mass of the lamina $(\bar{x},\bar{y})$ is defined by

$$\bar{x} = \frac{\iint_R x\,\rho(x,y)dxdy}{\iint_R \rho(x,y)dxdy}, \quad \bar{y} = \frac{\iint_R y\,\rho(x,y)dxdy}{\iint_R \rho(x,y)dxdy}$$

Write a program and execute it to calculate $(\bar{x},\bar{y})$ using Monte-Carlo method with the density function $\rho(x,y) = e^{-(x^2+y^2)}$ and the region described by $0 \le x \le 1.0$ *and* $0 \le y \le 1.0$. Use 1000 random numbers. [8]

Q.2. Under the Block-Gruneissen approximation for the resistance in a mono-valent metal, integrals of the following form need to be calculated;

$$\text{BGintegral} = \int_0^t \frac{x^5}{(e^x-1)(1-e^{-x})}dx$$

To avoid the singularity at x=0, assume lower limit of integration to be 0.001.

Write a program to calculate the integral using Simpson Rule for t=2 convergent to a precision of 0.001. This can be checked by initial estimation for some N steps and then with N+2 steps. If the difference between two estimates of the integral is small (in this case less than 0.001) then the integral is convergent. [15]
You may begin with N > 10.
Note: Use the algorithm given in the Appendix.

Q.3. A pendulum confined to a plane of length *L* and a point mass ***m*** is acted upon by driving force $f_d = f_0\cos(\omega_0 t)$ and a resistive force $f_r = -kv = -kL\frac{d\vartheta}{dt}$, where ***k*** reflects the strength of resistive force.

Writing $q = \frac{k}{m}$ and $b = \frac{f_0}{mL}$ and choosing *L/g =1.0*, we get

$$\frac{d^2\vartheta}{dt^2} + q\frac{d\vartheta}{dt} + \sin\vartheta = b\cos(\omega_0 t)$$

The second order differential equation is equivalent to two first order differential equations as given below:

$$\frac{d\vartheta}{dt} = \omega$$

$$\frac{d\omega}{dt} = -q\omega - \sin\vartheta + b\cos(\omega_0 t)$$

Assume initial boundary conditions : At $t = 0$, $\omega = 0$ and $\theta = -\pi$.

Write a procedure using the Euler method to calculate ω(t) and θ(t) in the interval $-\pi < \theta < \pi$ for any values of the parameters ω_0 , q and b . **[15]**

Note: Use the algorithm given in the Appendix.

SECTION B
(Practical Section)

Q.4. Consider the random walk problem in two dimensions on a square lattice. The walk starts at the origin which is at a lattice point approximately at the centre of the lattice and assume length of each step to be of one unit . Each point on the lattice is recognized by the coordinates ***(x,y)*** from the origin. At a distance **d=sqrt(x^2+y^2)** of 25 units, **in any direction**, there is a point of no return , that is, the walk terminates as soon as one reaches a distance of 25 units or more.

(a) Write a program and execute it to find the number of steps a walker would need to reach the point of no return starting from (x=0,y=0). **[25]**

Note: Consider a 200 x 200 matrix to begin with and if need be, increase the matrix dimension.

(b) Convert the program above **into a procedure** that can work for any matrix size **(n X n)** and any value of **d** for the point of no return. The output should have the number of lattice points that defines the square lattice, value of **d** for the point of no return and the number steps required to reach the point of no return.
Execute the procedure for the problem in (a) above. **[10]**

Use the uniform random number generator in the range (0,1) available in Maple.

Q.5. The differential equation for a non-linear harmonic oscillator with force due to drag=d v(t) , with $v(t) = \frac{dx(t)}{dt}$ and external force = a *cos(wt)* is given by the equation

$$\frac{d^2x(t)}{dt^2} + d\frac{dx(t)}{dt} + w_0^2\, x(t) + q\, x(t)^3 = a\cos(w\,t)$$

where d=drag co-efficient, w_0=strength of the linear term of the harmonic oscillator, q= strength of the non-linear term of the harmonic oscillator.

Assume the initial conditions: At t=0, $x(0) = 0$ *and* $\frac{dx(t)}{dt}\Big|_{t=0} = 1$.

(i) Taking the parameter values: w_0=1, w=1, q=1, a=1 and d=1, solve the differential equation numerically using the Maple commands for solving differential equations. **[5]**

Plot x(t) vs t for t=0..200 with the style = point. Comment. [5]
Plot v(t) vs x(t) for t=0..200. Comment. [5]
(ii) Replace the value a with a=5. Repeat the tasks of (i). Comment. [10]
(iii) Consider new parameter values: a=1 , d=0.05 and w=0.05. [10]
Repeat the tasks of (i). Comment.

@@@@END OF EXAMINATION@@@@

APPENDIX

1. Solution of First Order Differential Equation with initial Conditions:

The equation is of the form $\frac{dy}{dx} = f(x,y)$ with $y(x_0) = \alpha$.

(i) Euler's Method:

$$y_{i+1} = y_i + h\,f(x_i, y_i) \qquad \text{where } h = x_{i+1} - x_i$$

(ii) Fourth Order RK-Method:

$$y_{i+1} = y_i + (k_1 + 2k_2 + 2k_3 + k_4)/6$$

where

$$k_1 = h\,f(x_i, y_i)$$
$$k_2 = h\,f(x_i + 0.5h,\ y_i + 0.5k_1)$$
$$k_3 = h\,f(x_i + 0.5h,\ y_i + 0.5k_2)$$
$$k_4 = h\,f(x_i + h,\ y_i + k_3) \qquad \text{and} \quad h = x_{i+1} - x_i$$

2. Numerical Integration:

Simpson Rule:

$$\int_a^b f(x)dx = \frac{h}{3}\left[f(a) + f(b) + 4(f_1 + f_3 + f_5 + \ldots + f_{n-1}) + 2(f_2 + f_4 + f_6 + \ldots + f_{n-2})\right]$$

where $f_0 = f(a)$, $f_n = f(b)$, $f_1 = f(a+h)$, $f_2 = f(a+2h)$......etc. and n=even integer.