UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2006/2007

TITLE OF PAPER	:	ELECTROMAGNETIC THEORY
COURSE NUMBER	:	P331
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS	:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS
		EACH QUESTION CARRIES 25 MARKS
		MARKS FOR DIFFERENT SECTIONS OF EACH QUESTION ARE SHOWN IN THE RIGHT-HAND MARGIN

THIS PAPER HAS 14 PAGES, INCLUDING THIS PAGE. THIS INCLUDES 4 COPIES OF A GRID NEEDED TO ANSWER ONE OF THE QUESTIONS

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR

ELECTROMAGNETISM

Important equations required to describe the propagation of electromagnetic radiation in matter.

1 $\nabla.(\varepsilon\mathbf{E}) = \rho_f$

2 $\nabla.\mathbf{H} = 0$

3 $\nabla\mathbf{x}\mathbf{E} + \mu\dot{\mathbf{H}} = 0$

4 $\nabla\mathbf{x}\mathbf{H} - \varepsilon\dot{\mathbf{E}} = \mathbf{J}_f = \sigma\mathbf{E}$

$\nabla\mathbf{x}\nabla\mathbf{x}\mathbf{E} = -\nabla^2\mathbf{E} + \nabla(\nabla.\mathbf{E})$ cartesian co-ordinates

5 $\nabla^2\mathbf{E} - \mu\varepsilon\ddot{\mathbf{E}} - \mu\sigma\dot{\mathbf{E}} = 0$

6 $\nabla^2\mathbf{H} - \mu\varepsilon\ddot{\mathbf{H}} - \mu\sigma\dot{\mathbf{H}} = 0$

7 $\mathbf{E} = \mathbf{E}_0 \exp j(\omega t - kz)\,\mathbf{i}$

8 $\mathbf{H} = (k/\omega\mu)\mathbf{E}_0 \exp j(\omega t - kz)\,\mathbf{j}$

9 $-k^2 + \omega^2\varepsilon\mu - j\omega\sigma\mu = 0$

10 $k^2 = \varepsilon_r\mu_r k_0^2[1 - j\sigma/\omega\varepsilon]$

$k_0 = \omega/c$

$c = (\mu_0\varepsilon_0)^{-½}$

$\mathbf{S} = \mathbf{E}\mathbf{x}\mathbf{H}$ Poynting's vector

$E/H = \omega\mu/k$ wave impedance

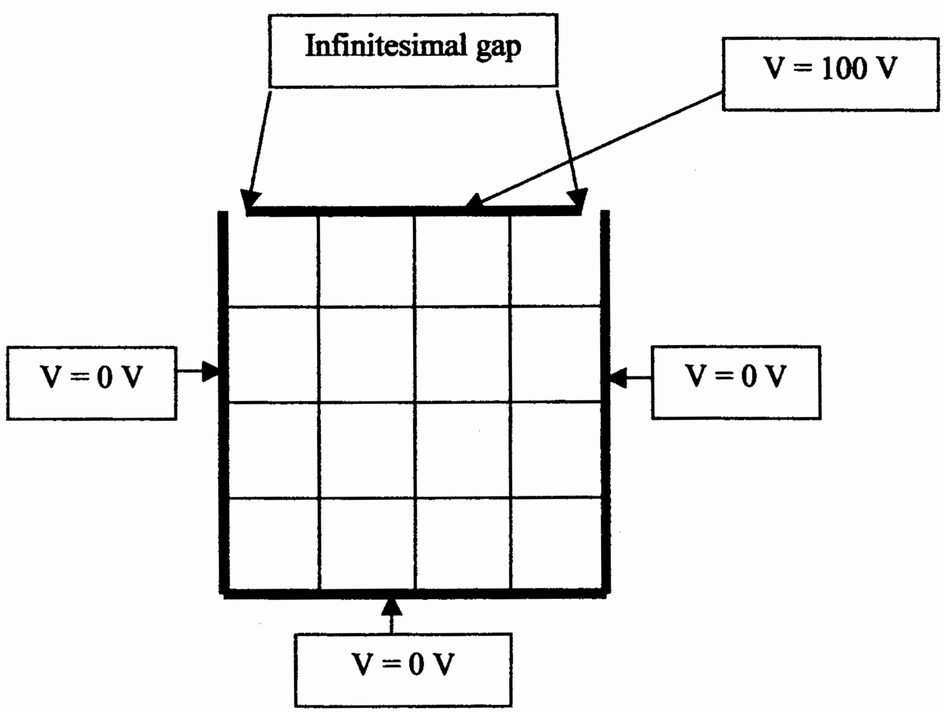
Question one

Show that the electrical potential at the centre of a sphere has the same value as the mean potential over the surface of the same sphere. [This is the so-called Mean-value theorem]. [10]

Earnshaw's theorem is concerned with the spatial variation of the magnitude of the potential in a region of space. State its content. [The theorem follows directly from the Mean-value theorem]. [5]

When the boundary conditions of an electrostatic problem are awkward, it is often very difficult to obtain an algebraic solution to either Poisson's or Laplace's equation. It is common practice to use a computer to find a numerical solution to the problem, based on the mean-value theorem.

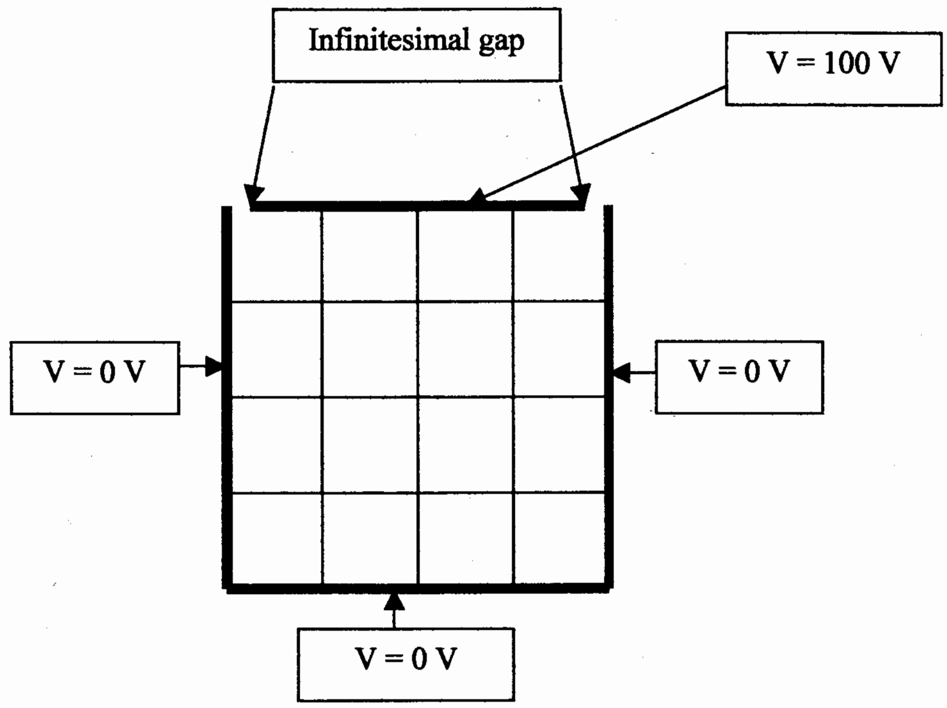
Consider a square region surrounded by conducting boundaries as shown in the figure.

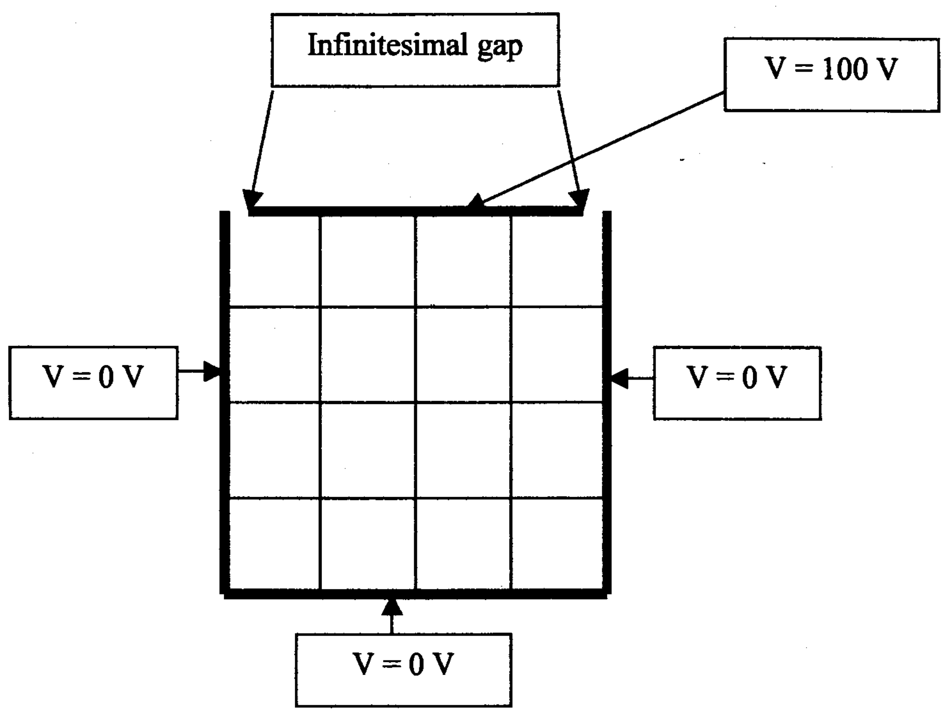


The potential of the top plate is 100 V while the other three conducting plates are at zero potential. Using an iteration method, determine the potentials at the grid points shown in the figure.

Perform at least two iterations. [10]

[You are provided with 4 copies of the electrode configuration and grid]


Infinitesimal gap
V = 100 V
V = 0 V
V = 0 V
V = 0 V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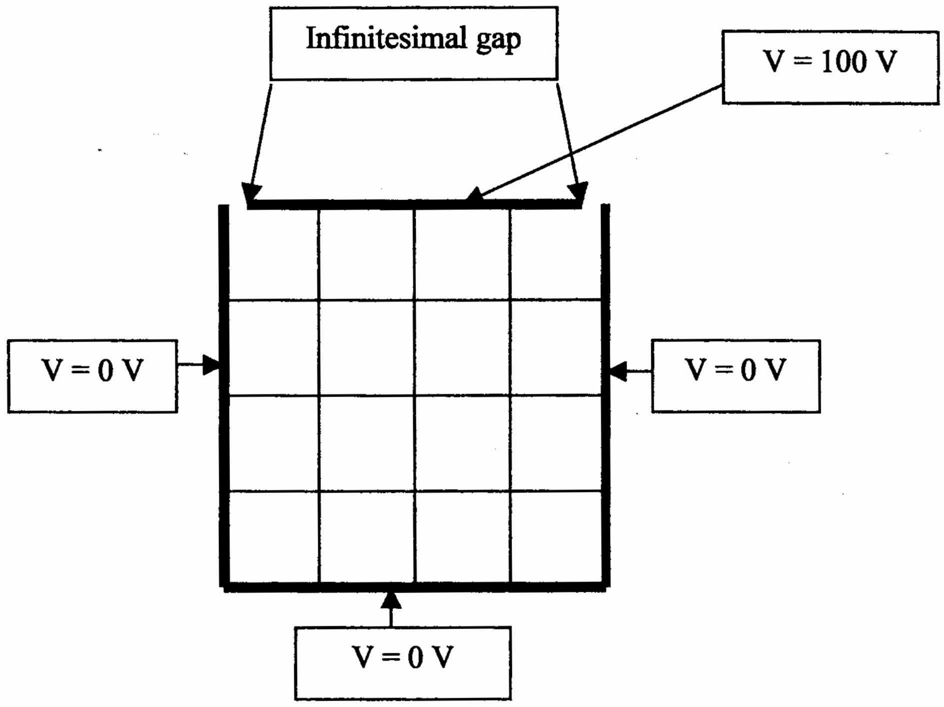
Infinitesimal gap
V = 100 V
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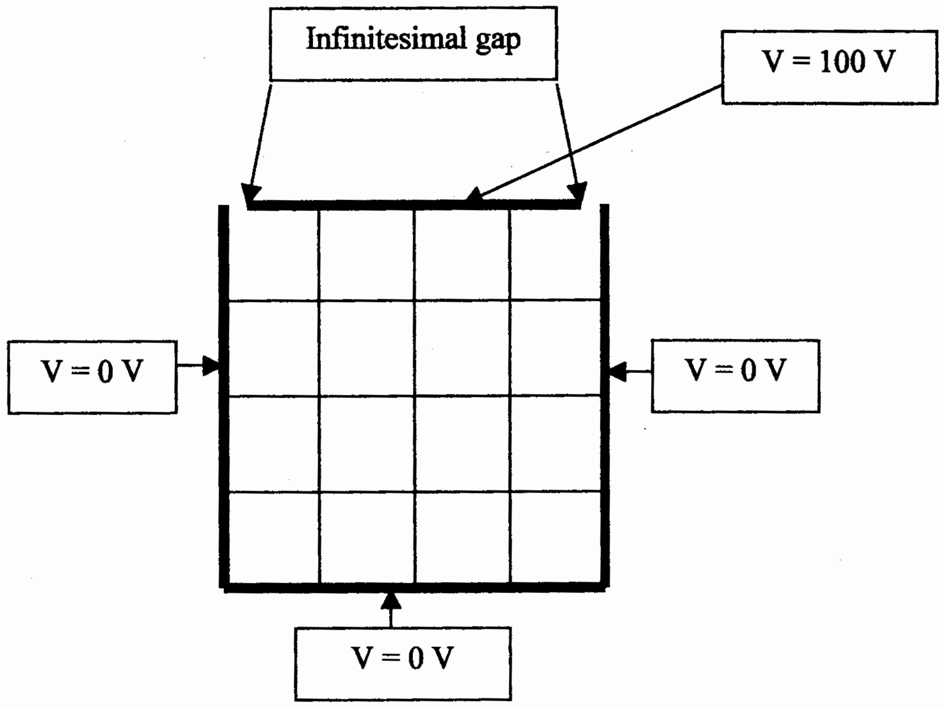

Infinitesimal gap
V = 100 V
V = 0 V
V = 0 V
V = 0 V

Question two

Write down a suitable expression for the electric field, $\boldsymbol{E}$, of a transverse electromagnetic wave propagating in free space and travelling in the z-direction. Explain what all the symbols mean in your expression for $\boldsymbol{E}$.

[5]

By manipulation of one of Maxwell's equations, given on the sheet "useful information" for this examination, or otherwise, show that the magnetic field, $\boldsymbol{H}$, of the wave is perpendicular to both $\boldsymbol{E}$ and z, and that its magnitude is given by

$$H = \frac{kE}{\omega\mu_0},$$

where the symbols have their usual meaning. [8]

Show that the instantaneous energy flux density of the wave in the direction of propagation has a magnitude of $c\varepsilon_0 E^2$. [6]

A laser beam has a mean power of 1 megawatt and a diameter of 1 millimetre. Calculate the *peak* value of E when the beam passes through a block of glass which is non-magnetic and has a refractive index of 1.4.

[6]

Question three

Show, using Maxwell's equations given on the sheet "useful information" for this examination, that for a linear, homogeneous, isotropic conductor

$$\nabla^2\mathbf{E} - \mu\varepsilon\ddot{\mathbf{E}} - \mu\sigma\dot{\mathbf{E}} = \nabla\left(\frac{\rho_f}{\varepsilon}\right), \qquad [1]$$

where all the symbols have their usual meanings. [5]

Show why the term on the right-hand side of the equation can be neglected when considering propagation of electromagnetic radiation, even in a conductor where $|\rho_f| \neq 0$. [5]

From consideration of equation [1], derive expressions for the skin depth in "good" and "bad" conductors. In the first case the displacement current can be neglected, while in the latter case, the term involving the conductivity may be assumed to be small compared with the displacement current.

Calculate the skin depth of an electromagnetic wave in sea water and copper at 1 kHz and at 10 GHz, given the following information:

	σ (Ω^{-1} m^{-1})	ε_r	μ_r
Cu	6×10^7	1.0	1.0
Sea water	5	70.0	1.0

[15]

Question four

Consider a possible solution to Maxwell's equations given by

$$\boldsymbol{A}(\boldsymbol{r},t) = \boldsymbol{A}_0 \exp j(\omega t - \boldsymbol{k}.\boldsymbol{r}) \qquad \varphi(\boldsymbol{r},t) = 0$$

where $\boldsymbol{A}$ is the vector potential and φ is the scalar potential. Assume that $\boldsymbol{A}_0$, $\boldsymbol{k}$ and ω are constants. Give, and interpret, the constraints on $\boldsymbol{A}_0$, $\boldsymbol{k}$ and ω imposed by each of Maxwell's equations [given on the page containing "useful information" for this examination]. [10]

Assume that no dielectrics or magnetic materials are present. What happens if the following changes are made?

a) If the signs of all the source charges are reversed, what happens to the electric and magnetic fields, $\boldsymbol{E}$ and $\boldsymbol{B}$? [5]

b) If the system is inverted in space, i.e. $\boldsymbol{r} \rightarrow -\boldsymbol{r}$, what happens to the charge and current densities, ρ and $\boldsymbol{J}$, and to $\boldsymbol{E}$ and $\boldsymbol{B}$? [5]

c) If the system undergoes time reversal, i.e. $t \rightarrow -t$, what changes occur to ρ, $\boldsymbol{J}$, $\boldsymbol{E}$ and $\boldsymbol{B}$? [5]

Question five

What is meant by the following terms:

The gauge for A

A retarded potential

The Hertzian dipole

Radiation resistance [8]

By using the Lorentz gauge, derive expressions for the field vectors, $\boldsymbol{E}$ and $\boldsymbol{H}$ in terms of the vector potential $\boldsymbol{A}$. [5]

In spherical polar co-ordinates, the components of the field vectors far from a Hertzian dipole are:

$$4\pi r E_\theta = j\,(\mu_0/\varepsilon_0)^{1/2}\,k\,Idl\,\sin\theta\,\exp(j\omega t')$$

$$4\pi r H_\phi = j\,k\,Idl\,\sin\theta\,\exp(j\omega t'),$$

where Idl is a current element, ω is angular frequency and t' is retarded time. Hence derive an expression for the time-averaged Poynting vector $|S_{av}|$ and show that it is radial away from the dipole. Sketch a plot of $|S_{av}|$ for a fixed distance about a current element placed at the origin and parallel to the z-direction. [12]