UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2006

TITLE O F PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

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QUESTION 1

(a) Prove the law of sines using the cross-product. **(5 marks)**

(b) A projectile is launched over a flat surface at an angle θ with the horizontal. Show that the maximum height is given by,

$$h = \frac{\upsilon_0^{\,2} \sin^2 \theta}{2g}, \text{ and}$$ **(7 marks)**

that the range is given by

$$R = \frac{\upsilon_0^{\,2} \sin 2\theta}{g}.$$ **(7 marks)**

(c) A wheel rotates at constant angular velocity $\dot{\theta} = \omega$ with its centre at the origin in the *x*-*y* plane in an anticlockwise direction (See Figure 1). At $t = 0$ a bead on one of the spokes is at the origin and the spoke is along the *x*-axis. The bead then moves at constant speed u radially outward along the spoke.

(i) What is the velocity of the bead in polar coordinates? **(3 marks)**

(ii) What is the velocity of the bead in Cartesian coordinates? **(3 marks)**

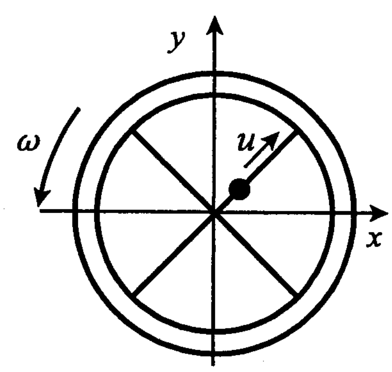

Figure 1.

QUESTION 2

(a) A ball of mass m falls from rest under gravity through a fluid which exerts a viscous resistive force which is equal to bv where v is the instantaneous velocity of the ball and b is a constant. Take the downward direction to be positive.

(i) Find the velocity of the ball as a function of time. **(4 marks)**

(ii) Sketch the velocity-time graph. **(2 marks)**

(iii) Does the ball reach terminal velocity? **(2 marks)**

(iv) Find the acceleration of the ball as a function of time. **(4 marks)**

(v) Sketch the acceleration-time graph. **(2 marks)**

(b) A bead of mass m with a hole has a wire going through the hole. The wire is bent into a circle and the bead can slide on the wire without friction. The wire loop spins in a vertical circle at constant angular velocity ω causing the bead to move around in a horizontal circle see Figure 2.

(i) Draw a clear force diagram for the bead from which useful equation can be obtained. **(4 marks)**

(ii) Find the angle θ in terms of R, ω and g. **(7 marks)**

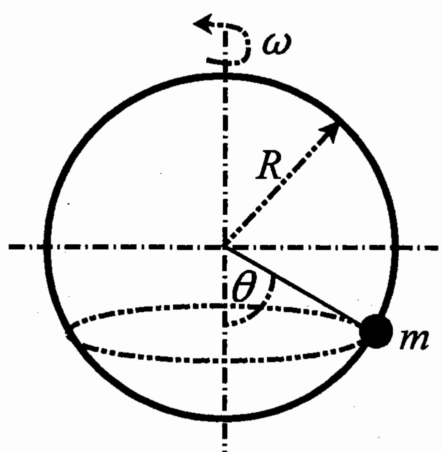

Figure 2.

QUESTION 3

(a) Find the centre of mass of a hollow hemisphere of mass M and inner radius R_1 *and outer radius* R_2 in terms of R_1, R_2 and the unit vector along z. Assume the hemisphere has its flat side as a base and the origin is at the centre of the sphere making the hemisphere. Start with the volume element

$$d\tau = r^2 dr\, \sin\theta\, d\theta\, d\phi.$$

The position vector in spherical coordinates is given by

$$\vec{r} = r\sin\theta\cos\phi\hat{i} + r\sin\theta\sin\phi\hat{j} + r\cos\theta\hat{k}.$$ **(10 marks)**

(b) An empty rail car of mass M_0 starts from rest under an applied force F. At the same time, maize begins to fill the car at a steady rate dm/dt b from a hopper at rest along the rail track (Figure 3). Find the velocity when a mass, m, of maize has been transferred to the rail car. The problem can be solved in only two steps, but use the *mass and momentum transport* method. Apply your solution to the case when M_0 = 500 kg, b = 20 kg/s and F = 100 N to find the velocity at time t = 10 s. **(12 marks)**

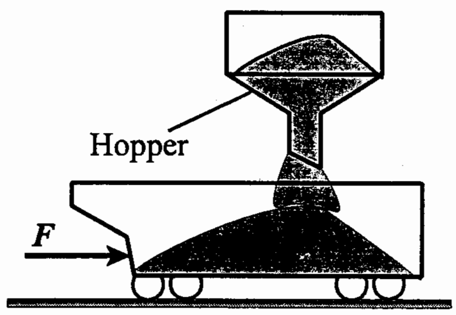

Figure 3.

(c) To reduce the force of impact with the ground, animals instinctively prolong the time of collision by lowering their centre of mass by a distance s when they fall to the ground. Show that the time for the collision is given by $t = \frac{2s}{\upsilon_o}$, where υ_0 is the incident velocity with the ground, and g is the acceleration due to gravity. **(3 marks)**

QUESTION 4

(a) A particle is given a horizontal velocity u at the top of a smooth sphere of radius R.

(i) If the particle follows the surface of the sphere, find the angle at which it leaves the sphere in terms of u, R and the gravitational acceleration g. **(9 marks)**

(ii) If the particle leaves the sphere at the highest point, determine the minimum value of u in terms of R and g. **(3 marks)**

(b) Show that a system with potential energy U and kinetic energy K of the following forms, respectively:

$$U = \frac{1}{2}Aq^2 + C,$$

$$K = \frac{1}{2}B\dot{q}^2,$$

oscillates harmonically, where A, B, and C are constants with appropriate units, and q is a variable appropriate to the particular problem. Also give then expression for the angular frequency. **(7 marks)**

(c) A particle of mass m moving in one-dimension along the x-axis is acted upon by a force directed toward the origin given by

$$F(x) = \frac{A}{x^2} + B,$$

where A and B are constants.

(i) Find the potential energy function. **(2 marks)**

(ii) Find the equilibrium position. **(2 marks)**

(iii) Determine the stability of the equilibrium point. **(2 marks)**

QUESTION 5

(a) A block of mass *m* slides freely in the positive *x*-direction as shown in Figure 4.

(i) Find the angular momentum about an axis through the origin *O*. **(3 marks)**

(ii) Find the angular momentum about ab axis through point *P*. **(4 marks)**

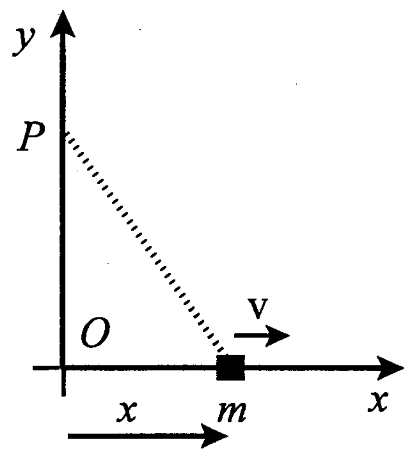

Figure 4.

(b) Now suppose that the block in (a) slides with friction.

(i) Find the torque about an xis through the origin. **(3 marks)**

(ii) Find the torque about an axis through point *P*. **(4 marks)**

(c) A uniform stick of length L is placed such that its left end is at the origin. Find its moment of inertia about an axis through the origin. **(5 marks)**

(d) In the problem in (c), what would be the moment of inertia a bout an axis through the origin if the centre of the stick coincides with the origin? **(6 marks)**