UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2006

TITLE O F PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE CHIEF INVIGILATOR

QUESTION 1

(a) Show that the volume of a parallelpiped object with edges $\vec{A}$, $\vec{B}$, and $\vec{C}$ is given by $\vec{A}\cdot(\vec{B}\times\vec{C})$, where the vectors $\vec{B}$ and $\vec{C}$ define the base of the parallelpiped and $\vec{A}$ is a vector giving the height, where A is B and C as shown in Figure 1. (See Figure 1).

(5 marks)

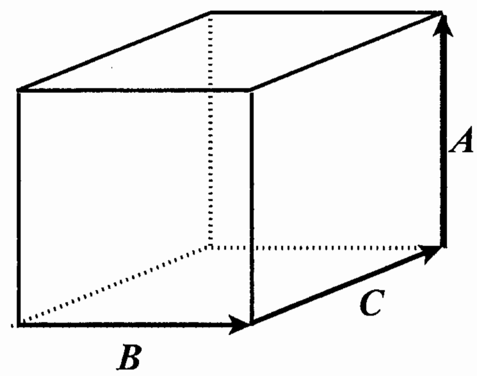

Figure 1.

(b) A projectile is fired up an inclined plane inclined at an angle ϕ with the horizontal, with a velocity υ_0 at an angle θ with the horizontal ($\theta > \phi$) (See Figure 2). Show that the projectile travels a distance d up the inclined plane, where

$$d = \frac{2\upsilon_0^{\,2}\cos\theta\sin(\theta-\phi)}{g\cos^2\phi}.$$

(8 marks)

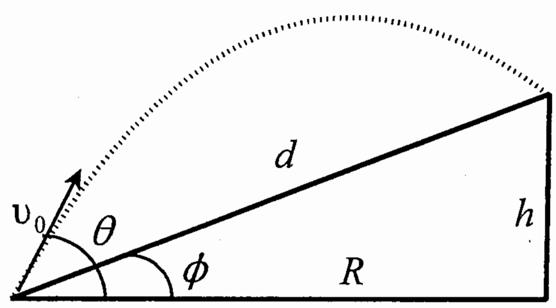

Figure 2.

(c) A particle is under the acceleration $a = g - \frac{k}{m}\upsilon$, where g, k, m are constants and υ is the instantaneous velocity of the particle.

(i) Find the velocity as a function of time. **(6 marks)**

(ii) Find the position y as a function of time. **(6 marks)**

QUESTION 2

(a) A particle of mass m slides without friction on the inside of a cone. The axis of the cone is vertical, and gravity is directed downward. The apex half angle of the cone is θ, as shown in Figure 3. The path of the particle is a horizontal circle and its speed constant and is υ_0.

(i) Draw a clear force diagram for the particle. **(4 marks)**

(ii) Find the radius of the circular path in terms of υ_0, g, and θ. **(7 marks)**

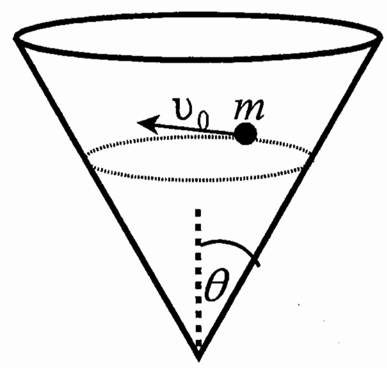

Figure 3.

(b) An automobile moving along a road enters a turn whose radius is R. The road is banked at an angle θ with the horizontal and the coefficient of friction between the wheels of the of the vehicle and the road is μ. Find expressions for the

(i) the minimum velocity and **(7 marks)**

(ii) maximum velocity for the vehicle to stay on the road without skidding in terms of R, μ, g, and θ. **(7 marks)**

QUESTION 3

(a) Find the centre of mass of a hemisphere of mass *M* and radius *R* in terms of *R* and the unit vector along z. Assume the hemisphere has its flat side as a base and the origin is at the centre of the sphere making the hemisphere. Start with the volume element $d\tau = r^2 dr\, \sin\theta\, d\theta\, d\phi$ The position vector in spherical coordinates is given by $\vec{r} = r\sin\theta\cos\phi\hat{i} + r\sin\theta\sin\phi\hat{j} + r\cos\theta\hat{k}$. **(10 marks)**

(b) A rocket ascends from rest in a uniform gravitational field by ejecting exhaust gases with constant speed *u* relative to the rocket. The rate at which mass is expelled by the rocket is $dm/dt = -dM/dt = \gamma M$, where *M* is the instantaneous mass of the rocket and γ is a constant. The rocket is also retarded by air resistance with a force of magnitude $Mb\upsilon$, where *b* is a constant and υ is the instantaneous velocity of the rocket.

(i) Find the velocity of the rocket as a function of time. **(13 marks)**

(ii) What is the terminal velocity of the rocket? **(2 marks)**

QUESTION 4

(a) A marble of mass m starts from rest at a height z and slides along a frictionless loop-the-loop track of radius R as shown in Figure 4. Use the work-energy theorem to find the the height z in terms of R so that at the top of the circular path of the track, the only force responsible for centripetal motion is the weight of the marble. **(9 marks)**

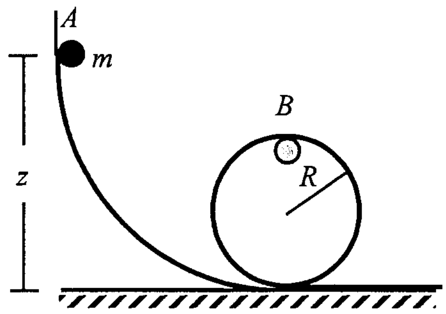

Figure 4.

(b) The Leonard Jones 6,12 potential for a two-atom system is given by

$$U = \varepsilon\left[\left(\frac{r_0}{r}\right)^{12} - 2\left(\frac{r_0}{r}\right)^{6}\right],$$

where ε and r_0 are positive constants and r is the instantaneous distance of separation.

(i) What is the force between the two particles? **(3marks)**
(ii) Find the equilibrium point. **(4 marks)**
(iii) Determine the stability of the equilibrium point. **(4 marks)**
(iv) What is the depth of the potential well? **(2 marks)**
(v) What is the angular velocity of small oscillations about the equilibrium point in terms of ε, r_0, and m if the two atoms have the same mass m? **(3 marks)**

QUESTION 5

(a) A uniform sphere of mass M and radius R and a uniform cylinder of the same mass M and radius R are released simultaneously from rest at the top of an inclined plane with friction. The inclined plane makes an angle θ with the horizontal. Find the acceleration of each body and determine which body arrives at the bottom first. The moment of inertia for the cylinder and sphere are $I_c = (1/2)MR^2$ $I_s = (2/5)MR^2$, respectively. **(9 marks)**

(b) A body of mass m is acted upon by a force due to a spring of spring constant k on a horizontal surface with grease that causes viscous friction $f = -b\upsilon$, where υ is the instantaneous velocity of the mass, and b is a constant. (Neglect the μN friction).

(i) Draw a force diagram for the mass and set up the equation of motion for the mass. Do not solve the equation. **(4 marks)**

(ii) Sketch a labeled graph of the motion of the mass if the solution is

$x = Ae^{-\frac{b}{2m}t}\cos(\omega t + \phi)$. **(4 marks)**

(iii) If the oscillator is now both damped and driven with a force $F_0 \sin \omega t$, make a force diagram for the mass and write down the equation of motion for the mass. **(4 marks)**

(iv) Sketch a labeled graph of the amplitude of the oscillation as a function of angular frequency for the cases $b = 0$, b small and b large for the problem in (iii). Also give the equation for the centre frequency ω_0. **(4 marks)**