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UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2005

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: P 461

TIME ALLOWED : THREE HOURS

ANSWER ANY FOUR QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS

VALUES OF SOME PHYSICAL CONSTANTS ARE GIVEN AT THE END OF THE PAPER

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THIS PAPER CONTAINS FIVE QUESTIONS

Question One

(a) (i) State what each of the following terms represent for a system of particles. macrostate , microstate and weight.

(3 marks)

(ii) A system consists of 5 bosons arranged in 3 energy levels each having degeneracy 4. What are the macrostates? Find the most probable configuration of the system.

[Given that the weight of a system of bosons, $W = \prod_{s} \frac{(g_s - 1 + n_s)!}{(g_s - 1)!n_s!}$]

(11 marks)

(b) (i) In question (a) (ii) above, the energies of levels 1,2 and 3 are 0 J, 2 J and 3 J respectively. If the total energy of the system is 10 J, find the occupation numbers of the energy levels. Does the system belong to the most probable configuration? (6 marks)

(ii) The allowed energies of a set of quantum particles is given as: $E = E_0 n^2$ where E_0 constant and $n^2 = n_x^2 + n_y^2 + n_z^2$, n_x , n_x , n_x being the quantum numbers representing an energy state. Given that the energy of a level is 26 E_0 find its degeneracy. (5 marks)

Question Two

(a) The partition function of a system Z is defined as $Z = \sum_s g_s e^{\beta\varepsilon_s}$. Derive the following expressions for the entropy and the total energy of a classical system:

(i) $$S = Nk \ln Z + \frac{E}{T}$$ (7 marks)

(ii) $$E = NkT^2 \frac{\partial}{\partial T} \ln Z$$ (6 marks)

(b) (i) Explain why the partition function $Z = \sum_s g_s e^{\beta\varepsilon_s}$

can be replaced by $Z = \int_0^\infty e^{-\varepsilon_s/kT} g(\varepsilon_s) d\varepsilon_s$

for a classical gas. (3 marks)

(ii) The partition function of a classical perfect gas, $Z = \frac{V}{h^3}(2\pi mkT)^{3/2}$.

Show that the entropy of a classical gas:

$$S = Nk \ln\left[\frac{V}{h^3}(2\pi mkT)^{3/2}\right] + \frac{3}{2}Nk$$

Comment on the validity of this result. (9 marks)

Question Three

(a) (i) Stating all the assumptions you have made, show that for the most probable configuration, the distribution of a system of bosons at temperature T can be represented as:

$$n_S = \frac{g_S}{e^{-(\alpha+\beta\varepsilon_S)} - 1}$$ where the symbols have their usual meanings.

[Given that the weight of a system of bosons, $W = \prod_S \frac{(g_S - 1 + n_S)!}{(g_S - 1)!n_S!}$]

(10 marks)

(ii) Show that at low density and at high temperature the above distribution function approximates to:

$$n_s = g_s \exp(\alpha + \beta\varepsilon_s)$$

Does this distribution function belong to the classical system? Why?

(7 marks)

[The multiplier α can be approximated to $\ln\frac{Nh^3}{V(2\pi mkT)^{3/2}}$]

(b) (i) State briefly what is meant by Bose-Einstein condensation.
Give an example. (5 marks)

(ii) Calculate the condensation temperature of a system of bosons from the following data:

volume of one mole of the gas = 2.7×10^{-5} m^3
mass of a particle = 6.65×10^{-27} kg.
Avogadro's number = 6×10^{23} / mol

[given $T_B = \frac{h^2}{2\pi mk}\left(\frac{N}{2.612V}\right)^{2/3}$] (3 marks)

Question Four

(a) The quantum statistical expression derived by Max Planck for the spectral distribution of energy from a black body is expressed as:

$$E(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT} - 1}$$

Derive the spectral distribution under short and long wavelength limits.

(8 marks)

(b) (i) Use the above Planck's distribution function to show that the total energy radiated is proportional the fourth power of the absolute temperature of the body. (See appendix for definite integrals) (10 marks)

(ii) Given that the proportionality constant in the above expression for total energy is equal to σ (4/c), where σ is the Stefan-Boltzmann constant, calculate the value of σ. (7 marks)

Question Five

(a) (i) Given that the density of states for a system of fermions:

$$g(\varepsilon)d\varepsilon = \frac{4\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$$, show that the Fermi energy of the system is:

$$\varepsilon_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$$, where symbols have their usual meanings.

(8 marks)

(ii) Use the above expression for the Fermi energy to show that the average kinetic energy of a particle in a Fermi gas at 0 K is (3/5) times the Fermi energy.

[Hint: average energy = $(1/N)\int n(\epsilon)d\epsilon$] (6 marks)

(b)

(i) Define the Fermi temperature T_F (2 marks)

(ii) Calculate T_F at 300 K for a metal with Fermi energy 3.12 eV.

(3 marks)

(iii) The electronic contribution to specific heat capacity is given as $C_V = 3NK\,T / T_F$. Comment on the effect of T_F as calculated in (ii) above on the specific heat .

(2 marks)

(c) Calculate the fermi energy of a metal (in electron volts) having an electron density of $5\text{x}10^{28}\ m^{-3}$. (4 marks)

Appendix A

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Plank's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	$1.67 \times 10^{-27 \text{ kg}}$
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan constant	σ	5.67×10^{-8} Wm^{-2}K^{-4}
Atmospheric pressure		1.01×10^{5} Nm^{-2}
Mass of $_2^4$ He atom		6.65×10^{-27} kg
Mass of $_2^3$ He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4×10^{-3} m^{3} mol^{-1}

Appendix B

Definite integrals.

$$\int_0^{\infty} e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^{\infty} e^{-ax^2} x^3\, dx = \frac{1}{2a^2}$$

$$\int_0^{\infty} e^{-ax^2} x^2\, dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^{\infty} x^{1/2} e^{-\lambda x}\, dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^{\infty} \frac{x^4 e^x}{(e^x - 1)^2}\, dx = \frac{4\pi^4}{15}$$

$$\int_0^{\infty} \frac{x^{1/2}}{e^x - 1}\, dx = \frac{2.61\pi^{1/2}}{2}$$

$$\int_0^{\infty} e^{-ax^2} x\, dx = \frac{1}{2a}$$

$$\int_0^{\infty} e^{-ax^2} x^5\, dx = \frac{1}{a^3}$$

$$\int_0^{\infty} e^{-ax^2} x^4\, dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^{\infty} \frac{x^3\, dx}{e^x - 1} = \frac{\pi^4}{15}$$