UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2005

TITLE OF THE PAPER: CLASSICAL MECHANICS

COURSE NUMBER : P320

TIME ALLOWED : THREE HOURS

INSTRUCTIONS:

ANSWER ANY **FOUR** OUT OF **FIVE** QUESTIONS.

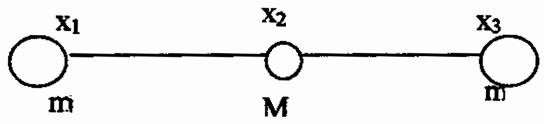
EACH QUESTION CARRIES **25** MARKS.
MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS **FIVE** PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THE PAPER UNTIL THE INVIGILATOR HAS GIVEN PERMISSION.

Q.1.

(a) The standard conformation of CO_2 molecule is a linear tri-atomic molecule. With mass of Oxygen as m and the mass of Carbon as M, the equilibrium configuration of the molecule is as shown in the figure below, assuming the vibrations are only along the line of molecule and are simple harmonic vibrations with spring constant k.
Equilibrium distance between each oxygen and the carbon is b .



Here x_1 , x_2 , x_3 are coordinates along the x-axis with respect to an arbitrary space fixed co-ordinate system.

(i) Define the centre of mass system. [2]

(ii) With the origin fixed at centre of mass, state the number of degrees of freedom the system has. Give an expression for the potential in terms of given co-ordinates. [5]

(b) A particle of mass m is described by the Lagrangian function ,

$$L=\frac{1}{2}m(\dot{x}^2+\dot{y}^2+\dot{z}^2)+\frac{m\omega}{2}(x\dot{y}-y\dot{x})$$

where ω is a constant angular frequency.

(i) Find the equations of motion. [6]

(ii) Show that z-component of momentum is a constant of motion. [2]

(iii) Define $r=x+iy$ and $q=x-iy$.

Show that the equations of motion reduce to [10]

$$\ddot{r}+i\omega\dot{r}=0 \quad , \ddot{q}-i\omega\dot{q}=0 \text{ and } \quad \ddot{z}=0 .$$

Q.2.

(a) A particle of mass m is under the influence of potential

$$V(r)=-\frac{k}{r}.$$

Assuming that the motion is planar, using plane polar coordinates r and θ,

(i) Derive an expression for the Lagrangian L in terms of polar coordinates r and θ. [5]

(ii) Show that $p_r=m\dot{r}$ and $p_\vartheta=mr^2\dot{\vartheta}$. [2]

Note: $p_i=\frac{\partial L}{\partial \dot{q}_i}$.

(iii) Using the definition for the Hamiltonian $H=\sum_i \dot{q}_i p_i - L$,

show that $H=\frac{p_r^2}{2m}+\frac{p_\vartheta^2}{2mr^2}-\frac{k}{r}$. [2]

(iv) which of the two expressions given above for L and H is expressed in terms of canonical variables. [1]

(v) Bounded orbits are possible only for the attractive potential and the orbits are ellipses or circles. [5]

(b) Keplar's 3rd law is given by the equation $\tau^2 = \frac{4\pi^2 m}{k} a^3$

where τ is the period of the orbit, m is the reduced mass and a is the semi-major axis .

The major axis of the elliptic orbit of a certain comet is 100 astronomical units.

(i) What is its period? [8]

(ii) What is the comet's speed at perihelion and aphelion? [2]

Given: $k = Gm_1m_2$,
G = Newtonian constant of gravitation= 6.673 x 10^{-11} $m^3kg^{-1}s^{-2}$
Mass of Sun = 1.99x10^{30} kg.
One astronomical unit (A.U.) = 1.495x10^{11} m.

Note: Assume that mass of sun is very large compared to the mass of comet.

Q.3.

(a) Explain the terms [6]

(i) Small oscillation.
(ii) Normal coordinates and normal vibrations.

(b) A system of one degree of freedom with potential energy *U(q)* is oscillating with small oscillations. Here q is the generalized coordinate.
Note that the potential *U(q)* is minimum at equilibrium.

If $\frac{d^2U}{dq^2} = k$, where k is a constant, show that the small oscillations of this system can be represented by a harmonic oscillator potential $\frac{1}{2} k x^2$ where x is the displacement of the system from equilibrium. [5]

(c) For a system of small oscillations, the Lagrange's equations are given as

$$(m+M)\ddot{z}_1 + m\ddot{z}_2 + \frac{k(2m+M)}{m} z_1 = 0$$

$$(m+M)\ddot{z}_2 + m\ddot{z}_1 + \frac{k(2m+M)}{m} z_2 = 0$$

(i) Show that the above equations can be reduced to equations for *normal vibrations* of following forms by defining appropriate co-ordinates q_1 and q_2 [5]

$$\ddot{q}_1 + \frac{k}{m} q_1 = 0$$

$$\ddot{q}_2 + \frac{k(2m+M)}{mM} q_1 = 0$$

(ii) Show that frequencies of normal vibrations are [4]

$$\varpi_1 = \sqrt{\frac{k}{m}} \quad \text{and} \quad \varpi_2 = \sqrt{\frac{k(2m+M)}{mM}}$$

(iii) Draw the diagrams of the two normal vibrations. [5]

Q.4.

(i) Show that for one dimensional motion of a particle of mass m with a constant acceleration α the Hamiltonian is given by the expression, [5]

$$H = \frac{p^2}{2m} - m\alpha x$$

(ii) evaluate the fundamental Poisson brackets *[x,H]* and *[p,H]* . [8]

(iii) For an equation of the type

$$\frac{du}{dt} = [u,H]$$

the formal solution is given by the series expansion

$$u(t) = u_0 + t[u,H]_0 + \frac{t^2}{2!}[[u,H],H]_0 + \frac{t^3}{3!}[[[u,H],H],H]_0 + ...$$

where subscript *0* denotes the initial conditions at *t=0*.

Use the above relation to show that for the given Hamiltonian, the complete solution
is given by

$$x = x_0 + \frac{p_0 t}{m} + \frac{\alpha t^2}{2}$$

where x_0 and p_0 are position and momentum at *t=0* . [12]

Q.5.

(a) To an observer in the rotating system, rotating with angular velocity ω , a moving particle is under the influence of an effective force

$$\vec{F}_{eff} = \vec{F} - 2m(\vec{\omega} \times \vec{V}_r) - m\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

where $\vec{F}$ = force in the inertial system,
$\vec{V}_r$ = velocity of the particle in the rotating set of axes.

(i)Which is the term in the above expression corresponding to Coriolis force? [1]

(ii)What is the physical interpretation of the last term? [1]

(iii)Earth rotates with $\omega = 7.292 \times 10^{-5}$ *rad/s*. Calculate the value of centripetal acceleration at the equator where $r = 6.378 \times 10^{6}$ *m*.
Explain the effect of this acceleration on a falling body. [3]

(iv)At the Northern Hemisphere, what is the direction of ω . [5]
Fixing the z-axis along this direction, diagrammatically illustrate the deflection of a projectile shot along the earth's surface.

(v)What will be the direction of the projectile if shot along the earth's surface at Southern Hemisphere and at the Equator? [2]

(b) (i) A rigid body in space needs at most only six generalized coordinates to specify its configuration. Explain. [3]

(ii)Given that

$$\frac{d\vec{L}}{dt} + \vec{\omega} \times \vec{L} = \vec{N}$$

where $\vec{L} = \vec{I} \cdot \vec{\omega}$ = total angular momentum.

$\vec{\omega}$ = angular velocity .

$\vec{N}$ = moment of force or torque.

Assuming there are no external forces, show that for any rigid body , if initially the body is rotating about one of the principal axis, it will continue to rotate with constant angular velocity. [10]

@@@@END OF EXAMINATION@@@@