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UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2005

TITLE OF THE PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: Answer four Questions. Each Question carries 25 Marks. Marks for each section are as shown

Do not open this Paper until you told to do so by the **Invigilator**.

This Paper contains **SEVEN PAGES** including this one. There are also two pages of data

Question 1

a) Explain what happens when a moving negative charge enters a region of space in which the electric field is directed vertically upward. (3 Marks)

b) Fig 1.1 shows a plastic rod having a uniformly distributed charge -Q. The rod has been bent into a 120-degree circular arc of radius r. The coordinate axes are placed such that the axis of symmetry of the rod lies along the x-axis. The origin is at centre of the curvature P of the rod.

(i) Show that the vertical component of the electric field vanishes. (4 Marks)

(ii) In terms of Q and r what is the electric field E due to the rod at point P. (9 Marks)

c) A charge $q_1 = 7\mu C$ is located at the origin of a Cartesian coordinate system and a second charge $q_2 = -5\mu C$ is located on the x-axis 0.3m from the origin. Show the locations of the charges and the electric field vector in relation to a point P which is on the y-axis with coordinates (0,0.4)m. Find the numerical value of the electric field vector at point P.

(9 Marks)

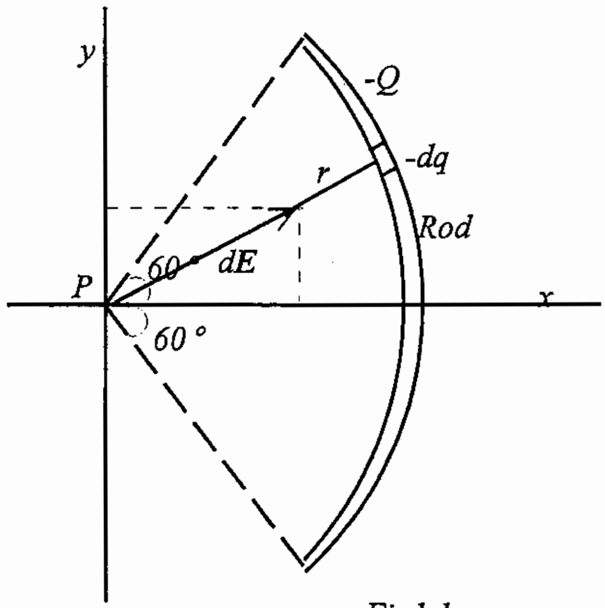

Fig1.1

Question 2

a) (i) A deuteron (a nucleus which consists of one proton and one neutron) is accelerated through a potential difference of 2.7kV. How much energy does it gain? What is the maximum speed if it started from rest? (5 Marks)

(ii) The electric potential due to a small charged sphere can be given as

$$\varphi(r) = k\frac{q}{r}$$

where all symbols have their usual meaning.
Deduce the electric field. Give the answer in terms of the unit vectors **i**, **j** and **k**.. (5 Marks)

b) A uniformly charged disk of radius a and charge density σ is positioned such that its axis is parallel and coincident with the x-axis. A point of observation P lies at a distance x along the x-axis.

(i) Sketch the arrangement and indicate all the relevant information needed to answer (ii) and (iii). (2 Marks)

(ii) Find the electrical potential at point P due to a small charge distribution dq on the disk. (4 Marks)

(iii) What is the electrical potential due to the total charge on the disk? (5 Marks)

c) In Fig 2.1, three long lines of charge, with the same sign, magnitude , and distribution, extend through the plane of the page. Sketch the electric field lines in that plane.

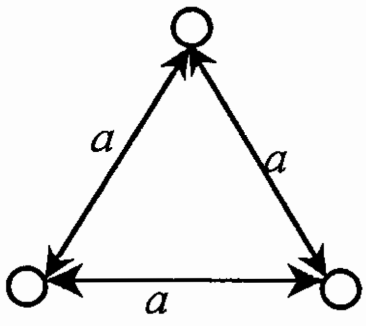

Fig 2.1

(4 Marks)

Question 3

a) (i) Two wires A and B of circular cross section are made of the same metal and are of equal lengths, but the resistance of wire A is three times greater than that of wire B. What is the ratio of their cross sectional areas? How do their radii compare? (6 Marks)

(ii) The current I (in A) in a conductor varies with time as

$$I = 2t^2 - 3t + 7$$

where t is in s. What quantity of charge moves across a section through the conductor during the interval t=2s and t=5s? (5 Marks)

b) (i) Starting with the definition of temperature coefficient of resistivity α deduce how resistivity ρ varies with temperature. (4 Marks)

(ii) A cylindrical metal rod is 1.60m long and 5.50mm in diameter. The resistance between its two ends (at 20°C) is $1.09\text{x}10^{-3}\Omega$. What is the material? A round disk is formed of the same material. What is the resistance between the round faces, assuming that each face is an equipotential surface? The disk has diameter 2.00cm and thickness 1.00mm. (7 Marks)

(iii) A colour code makes it possible to determine the value of a given resistor including its tolerance. What multipliers are associated with the colours black and red and what tolerance with the colour gold? (3 Marks)

Question 4

a) Consider the circuit shown in Fig 4.1. The curved segments are parts of circles of radius r_1(=0.2m) and r_2(=0.4m).The straight segments are along the radii. The angle θ is $2\pi/3$ radians. Find the magnetic field B at P assuming a current of 3A is flowing in the circuit. (8 marks)

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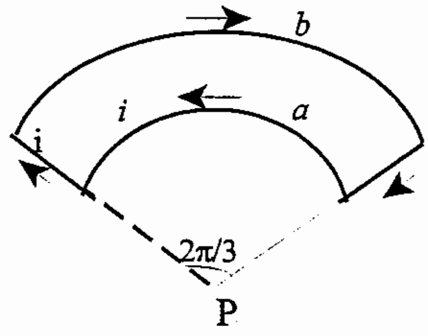

Fig 4.1

b) A toroid has a 200-turn winding carrying a current of 300A. The inner and outer radii are 0.4m and 1.2m respectively.

a) Draw the geometry of the problem. (2 marks)

b) Use Ampere's law to find the magnetic field at the inner and outer radii (6 marks)

c) The wire shown in Fig 4.2 carries current i. What magnetic field **B** is produced at the centre C of the semi-circle by:

(i) each straight segment of length L? (4 marks)

(ii) the semi-circular segment of radius R? (4 marks)

(iii) the entire wire? (1 mark)

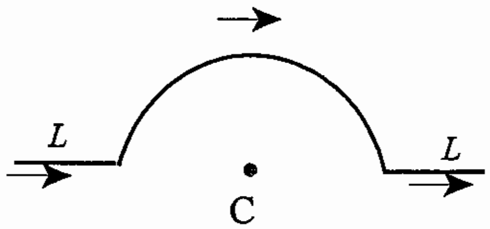

Fig 4.2

Question 5

a) A long solenoid of radius R has n turns per unit length and carries a time-varying current that varies sinusoidally as:

$$i = I_0 \cos\omega t$$

where I_0 is the maximum current and ω in the natural frequency of the current source.

(i) Sketch the geometry of the problem. (2 marks)

(ii) Determine the induced electric field outside the solenoid, a distance r from its axis. (6 marks)

(iii) What is the induced electric field inside the solenoid, a distance r from the axis. (4 marks)

(iv) For (ii) and (iii), sketch how the field varies, in each case, both in time and position. (8 marks)

b) The current in a solenoid is increasing at the rate of 10A/s. The cross-sectional area of the solenoid is π cm^2 and there are 300 turns on its 15-cm length. What is the induced emf which acts to oppose the increasing current? (5 marks)

Question 6

a) An oscillating LC circuit consisting of a 1.0-nF capacitor and a 3.0-mH coil has a peak voltage of 3.0V.

(i) What is the maximum charge on the capacitor? (4 marks)

(ii) What is the peak current through the circuit? (5 marks)

(iii) What is the maximum energy stored in the magnetic field of the coil? (3 marks)

b) Consider the circuit in Fig 6.1. With switch S_1 closed and the other two switches open, the circuit has time constant τ_C . With switch S_2 closed and the other two switches

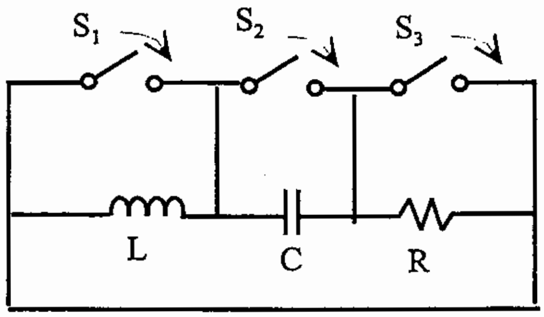

Fig 6.1

open, the circuit has time constant τ_L. With switch S_3 closed and the other two switches open, the circuit oscillates with a period T. Show that

$$T = 2\pi\sqrt{\tau_C \tau_L}$$ (13 marks)

Some Fundamental and Other Constants

Deuteron mass	m_d	3.34×10^{-27} kg
Electron mass	m_e	9.11×10^{-31} kg
Electron charge	e	1.6×10^{-19} C
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Wb/m^2
Permittivity of free space	ϵ_0	8.85×10^{-12} C^2/N.m^2

Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	**Resistivity** (Ω.m)	**Temperature Coefficient** $\alpha[(C°)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminium	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinium	11×10^{-8}	3.92×10^{-3}

Some Integrals

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ell n\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \frac{xdx}{\sqrt{a^2 - x^2}} = -\sqrt{a^2 - x^2}$$

$$\int \frac{xdx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$\int \sin ax dx = -\frac{1}{a}\cos ax$$

$$\int \cos ax dx = \frac{1}{a}\sin ax$$

Some Fundamental Constants[a]

Quantity	Symbol	Value[b]
Atomic mass unit	u	$1.660\ 540\ 2(10) \times 10^{-27}$ kg $931.434\ 32(28)$ MeV/c^2
Avogadro's number	N_A	$6.022\ 136\ 7(36) \times 10^{23}$ (g mol)$^{-1}$
Bohr magneton	$\mu_B = \frac{e\hbar}{2m_e}$	$9.274\ 015\ 4(31) \times 10^{-24}$ J/T
Bohr radius	$a_0 = \frac{\hbar^2}{m_e e^2 k}$	$0.529\ 177\ 249(24) \times 10^{-10}$ m
Boltzmann's constant	$k = R/N_A$	$1.380\ 658(12) \times 10^{-23}$ J/K
Compton wavelength	$\lambda_C = \frac{h}{m_e c}$	$2.426\ 310\ 58(22) \times 10^{-12}$ m
Deuteron mass	m_d	$3.343\ 586\ 0(20) \times 10^{-27}$ kg $2.013\ 553\ 214(24)$ u
Electron mass	m_e	$9.109\ 389\ 7(54) \times 10^{-31}$ kg $5.485\ 799\ 03(13) \times 10^{-4}$ u $0.510\ 999\ 06(15)$ MeV/c^2
Electron-volt	eV	$1.602\ 177\ 33(49) \times 10^{-19}$ J
Elementary charge	e	$1.602\ 177\ 33(49) \times 10^{-19}$ C
Gas constant	R	$8.314\ 510(70)$ J/K·mol
Gravitational constant	G	$6.672\ 59(85) \times 10^{-11}$ N·m^2/kg^2
Hydrogen ground state	$E_0 = \frac{m_e e^4 k^2}{2\hbar^2} = \frac{e^2 k}{2a_0}$	$13.605\ 698(40)$ eV
Josephson frequency-voltage ratio	$2e/h$	$4.835\ 976\ 7(14) \times 10^{14}$ Hz/V
Magnetic flux quantum	$\Phi_0 = \frac{h}{2e}$	$2.067\ 834\ 61(61) \times 10^{-15}$ Wb
Neutron mass	m_n	$1.674\ 928\ 6(10) \times 10^{-27}$ kg $1.008\ 664\ 904(14)$ u $939.565\ 63(28)$ MeV/c^2
Nuclear magneton	$\mu_n = \frac{e\hbar}{2m_p}$	$5.050\ 786\ 6(17) \times 10^{-27}$ J/T
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2 (exact)
Permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	$8.854\ 187\ 817 \times 10^{-12}$ C^2/N·m^2 (exact)
Planck's constant	h $\hbar = h/2\pi$	$6.626\ 075(40) \times 10^{-34}$ J·s $1.054\ 572\ 66(63) \times 10^{-34}$ J·s
Proton mass	m_p	$1.672\ 623(10) \times 10^{-27}$ kg $1.007\ 276\ 470(12)$ u $938.272\ 3(28)$ MeV/c^2
Quantized Hall resistance	h/e^2	$25812.805\ 6(12)$ Ω
Rydberg constant	R_H	$1.097\ 373\ 153\ 4(13) \times 10^7$ m^{-1}
Speed of light in vacuum	c	$2.997\ 924\ 58 \times 10^8$ m/s (exact)

[a] These constants are the values recommended in 1986 by CODATA, based on a least-squares adjustment of data from different measurements. For a more complete list, see Cohen, E. Richard, and Barry N. Taylor, *Rev. Mod. Phys.* 59:1121, 1987.

[b] The numbers in parentheses for the values below represent the uncertainties in the last two digits.