23

UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS AND ELECTRONIC ENGINEERING

SUPPLEMENTARY EXAMINATION 2005

TITLE O F PAPER:	MECHANICS
COURSE NUMBER:	P211
TIME ALLOWED:	THREE HOURS
INSTRUCTIONS:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS
	EACH QUESTION CARRIES 25 MARKS
	MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE CHIEF INVIGILATOR

QUESTION 1

(a) Prove the law of sines using the cross product. **(6 marks)**

(b) Write down the vector $\vec{r}$ to a point P in spherical coordinates, and show how it is obtained from the Cartesian coordinates using a clear diagram. **(5 marks)**

(c) Find the volume of a hollow hemisphere of inner radius R_1 and outer radius R_2. **(5 marks)**

$d\tau = r^2 dr \sin\theta d\theta d\phi.$

(d) Given that

$$\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}, \text{ and that}$$

$$\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r},$$

find the components of the velocity and acceleration of a particle in plain polar coordinates if the position vector is given by

$$\vec{r} = r\hat{r}.$$

Also state the meaning of each term in the acceleration. **(9 marks)**

QUESTION 2

(a) A body of mass m moves freely in the positive x-direction with a velocity v_0. When it reaches $x = 0$ it encounters a surface with a velocity dependent friction $f = Cv$, where C is a constant and v is the instantaneous velocity of the mass.

(i) Start with a force diagram for the mass and find the velocity as a function of time. **(4 marks)**

(ii) Find the terminal velocity and sketch the velocity-time graph. **(3 marks)**

(iii) Find the position as a function of time and sketch the distance-time graph. **(5 marks)**

(b) A mass m is attached by two strings to a vertical rod, as shown in Figure 1. The entire system rotates with constant angular velocity ω about the axis of the rod. Assume that ω is large enough to keep both strings taut, making the lower string of length R to be horizontal and the upper string to make an angle θ with the rod.

(i) Make a complete force diagram for the mass m. **(3 marks)**

(ii) Find the tension in each string in terms of m, g, R, and θ. **(6 marks)**

(iii) What is the minimum angular velocity ω_{min} for the lower string to be just taut. **(4 marks)**

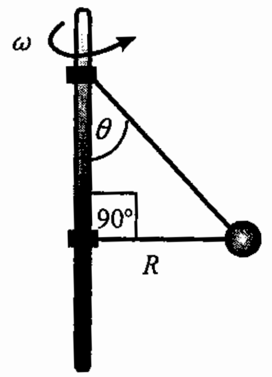

Figure 1.

QUESTION 3

(a) A rod of length L has a non-uniform length density λ, where $\lambda = \lambda_0 x/L$, where λ_0 is a constant and x is the distance from the less dense end of the rod. Find the centre of mass of the rod in terms of L and $\hat{i}$ assuming that the less dense end of the rod is at the origin and the rod is lying along the x-axis. **(6 marks)**

(b) A cannon of mass M fires a projectile of mass m at a velocity v_0 directed at an angle θ with the horizontal. Find an expression for the recoil speed V' the cannon if M = 5000 kg, m = 200 kg, v_0 = 125 m/s and θ = 40°. First find the general expression for the recoil velocity. **(7 marks)**

(c) Derive an expression for the final speed v_f of a rocket lifting off from a gravitational field g in terms of the speed of the ejected exhaust gases u, the initial mass of the rocket M_0, the final mass of the rocket M_f, the graviational acceleration g, and the time taken t. **(12 marks)**

QUESTION 4

(a) A small sphere of mass m slides down a circular path of radius R cut into a large block of mass M, as shown in Figure 2. The mass M rests on a frictionless table and the circular path cut in the larger block is frictionless. Both bodies are initially at rest and the mass m starts from the top of the path. Find the velocity with which the cube leaves the block at the bottom of the path. **(7 marks)**

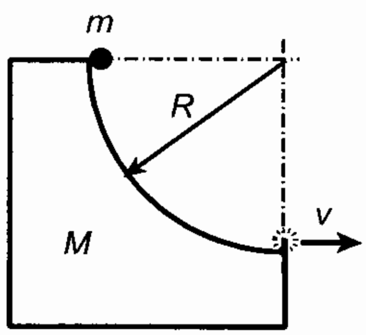

Figure 2.

(b) A smooth hemispherical bowl of internal radius R is fixed with its rim horizontal and uppermost. A marble of mass m is placed on the inner surface of the bowl at a point which is $R/2$ above the lowest point of the bowl. It is given an initial velocity $v_0 = (Rg/3)^{1/2}$ so that it initially moves down the surface of the bowl in a vertical circle. Figure 3 will be helpful in this problem.

(i) Use the work-energy theorem to determine the maximum height h reached by the marble on the other side in terms of the radius R. **(7 marks)**

(ii) Use the work-energy theorem to determine a general expression for the velocity of the marble in terms of g, R, and θ. **(7 marks)**

(iii) Find the normal force on the marble when it reaches the lowest point in the bowl in terms of m and g. **(4 marks)**

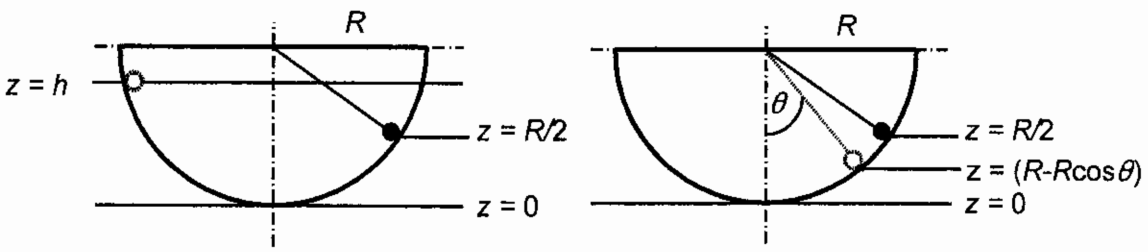

Figure 3. (a) (b)

QUESTION 5

(a) Show that when a particle has potential energy U and kinetic energy K of the form

$$U = \frac{1}{2}Aq^2 + C, \text{ and}$$

$$K = \frac{1}{2}B\dot{q}^2,$$

where q is a variable appropriate to the problem, and A, B, and C are constants, the particle oscillates harmonically with angular frequency $\omega = \sqrt{A/B}$. **(6 marks)**

(b) A conical pendulum has a string of length l attached at point B making an angle α with the vertical (see Figure 4).

Find the angular momentum for a conical pendulum at

(i) point A which is the centre of the horizontal circle of radius r in which the mass m moves, and **(6 marks)**

(ii) point B where the pendulum string is fixed and make a diagram that illustrates the angular momentum at this point. **(8 marks)**

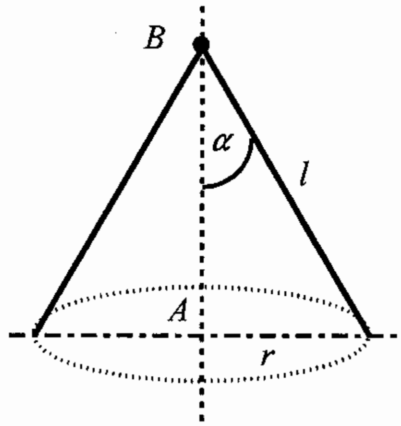

Figure 4.

(c) Show that the torque $\vec{\tau}$ and angular momentum $\vec{L}$ are related by the following equation:

$$\vec{\tau} = \frac{d\vec{L}}{dt}.$$ **(5 marks)**