University of Eswatini

Final Examination, December 2019

M.Sc MATH, M.Sc MID

Title of Paper : Environmental Fluid Mechanics

Course Code : MAT631

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SEVEN questions answer ANY FIVE questions.
2. Each question worth 20%.
3. Show all your working.
4. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) The velocity components in a steady, incompressible, two-dimensional flow field are $u = 2y$, $v = 4x$.

(i) Determine the corresponding stream function. [6]

(ii) Show on a sketch streamlines. Indicate the direction of flow along the streamlines. [6]

(b) What is the irrotational velocity field associated with the velocity potential $\phi = 3x^2 - 3x + y^2 + 16t^2 + 12zt$? Does the flow field satisfy the incompressible continuity equation? [8]

Question 2

(a) For the velocity field $V = 2xy\hat{i} + 4tz^2\hat{j} - yz\hat{k}$, find

(i) the acceleration. [6]

(ii) the angular velocity about the z-axis and the vorticity vector at the point (2, -1, 1) at t=2. [6]

(b) Consider velocity field with components $u = cx + 2\omega_0 y + u_0, v = cy + v_0, w = -2cz + \omega_0$, where c, u_0, v_0 and w_0 are constants. Determine the different types of motion, (translation, rotation, linear strain and shear stress), involved. [8]

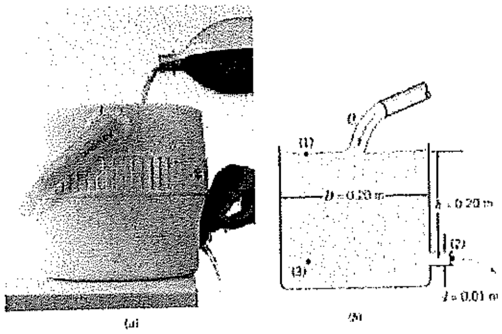
Question 3

(a) For a flow with velocity field $u = \dfrac{x}{1+t}$, $v = \dfrac{y}{1+t}$, $w = \dfrac{z}{1+t}$, find

(i) the streamline. [6]

(ii) pathline of particle of the particles. [6]

(b) A stream of refreshing beverage of diameter $d = 0.01m$ flows steadily from the cooler of diameter $D = 0.20m$ as shown in Figs. (a) and (b). Determine the flow-rate, Q, from the bottle into the cooler if the depth of beverage in the cooler is to remain constant at $h = 0.20m$. [8]



Question 4

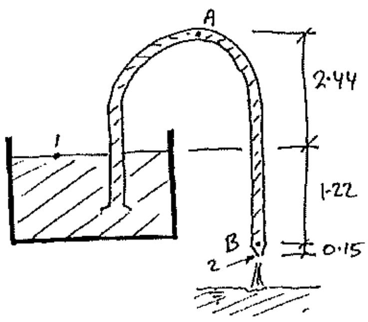
(a) A useful theoretical equation for computing the relation between the pressure, velocity, and altitude in a steady flow of a nearly inviscid, nearly incompressible fluid is the Bernoulli relation, named after Daniel Bernoulli

$$\frac{P}{\rho g} + \frac{1}{2g}V^2 + z = C.$$

(i) Show that the above equation satisfies the principle of dimensional homogeneity. [6]

(ii) Determine the dimension of the constant. [4]

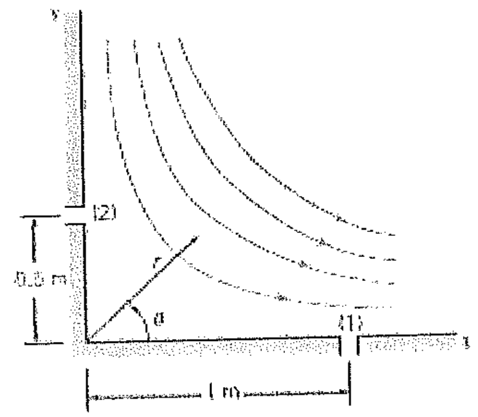
(b) For the siphon shown, determine the discharge given that the pipe diameter is 200 mm and the nozzle diameter is 150 mm. You may neglect friction in the pipe. [10]



Question 5

(a) If a stream function of a flow is given as $\Psi = A\theta$, determine the potential function ϕ. [6]

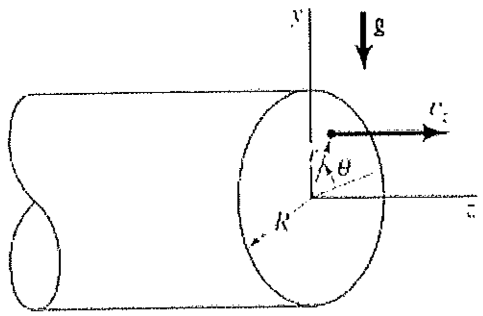
(b) The two-dimensional flow of a nonviscous, incompressible fluid in the vicinity of the 90^0 corner of the figure below is described by the stream function $\Psi = 2r^2 \sin\theta$ where Ψ has units of m^2/s when r is in meters. Assume the fluid density is $10^3\ Kg/m^2$ and the x–y plane is horizontal that is, there is no difference in elevation between points (1) and (2).



(i) Determine, if possible, the corresponding velocity potential. [6]

(ii) If the pressure at point (1) on the wall is 30 kPa, what is the pressure at point (2)? [8]

Question 6



(To find an exact solution for a laminar incompressible steady flow in a circular pipe). Assume the flow is

- steady
- constant density (incompressible)
- Newtonian fluid
- fully-developed
- laminar and parallel to the wall
- axisymmetric with no swirl
- zero velocity on the wall (no slip condition).

(i) Show the flow satisfies the continuity equation. [3]

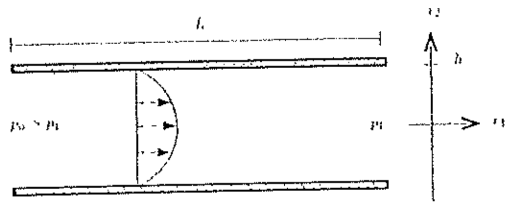
(ii) Formulate the momentum equation for this flow. [5]

(iii) Determine the velocity of the flow. [6]

(iv) Determine the flow rate. [3]

(v) Determine wall shear stress. [3]

Question 7



(Plane Pouseuille flow-exact solution for channel flow). Assume that the flow is

- inside the two-dimensional channel (x_1, x_2),
- driven by a pressure difference $p_0 - p_1$ between the inlet and the outlet of the channel,
- steady,
- constant density (incompressible),
- Newtonian fluid,
- fully-developed,
- zero velocity on the wall (no slip condition).

(i) Show that the flow satisfies the continuity equation. [3]

(ii) Formulate the momentum equation for this flow. [6]

(iii) Determine the velocity of the flow. [6]

(iv) Determine the flow rate. [5]

End of examination paper

If required:

$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}-\frac{u_\theta^2}{r}+u_z\frac{\partial u_r}{\partial z}\right)=$$

$$-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r u_r\right)\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right]+F_r$$

$$\rho\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{u_r u_\theta}{r}+u_z\frac{\partial u_\theta}{\partial z}\right)=$$

$$-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r u_\theta\right)\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right]+F$$

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right)=$$

$$-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]+F_z$$