UNIVERSITY OF SWAZILAND

MAIN EXAMINATION, 2019/2020

MSc.I

Title of Paper : Special Topics In Financial Mathematics

Course Number : MAT622

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions.
2. , each worth 25%. Answer ANY FOUR (4) questions.
3. Show all your working.
4. Start each new major question on a new page and clearly indicate the question number at the top of the page.
5. You can answer questions in any order.
6. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

ANSWER ANY *FOUR* QUESTIONS

QUESTION A1 [*25 Marks*]

A1 (a) Prove the following identities

i. $\ddot{a}^{(m)}_{\overline{n}|} = \frac{1-v^n}{d^{(m)}}$

ii. $I\ddot{a}_{\overline{\infty}|} = I^{(1)}\ddot{a}^{(1)}_{\overline{\infty}|} = \frac{1}{d^2}$

[6 Marks]

(b) A loan of E400,000 is being repaid by a 30−year increasing annuity immediate. The initial payment is k, and each subsequent payment is k larger than the preceding payment. The annual effective interest rate is 4%. Calculate the principal outstanding immediately after the ninth payment.

[9 Marks]

(c) The death benefit on a life insurance policy can be paid in four ways. All payments have the same present value:

(i) A perpetuity of E1200 at the end of each month, first payment one month after the moment of death;

(ii) Payments of E3654.70 at the end of each month for n years, first payment one month after the moment of death;

(iii) A payment of E178,663.20 at the end of n years after the moment of death; and

(iv) A payment of X at the moment of death.

Calculate X.

[10 Marks]

QUESTION A2 [*25 Marks*]

A2 (a) Given:

$$_tp_x = 1 - \left(\frac{t}{100}\right)^{1.5}$$

for $x = 60$ and $0 < t < 100$. Calculate $E(T(x))$.

[6 Marks]

(b) Mortality follows de Moivre's law and $E[T(16)] = 36$. Calculate $Var(T(16))$.

[8 Marks]

(c) Consider two independent lives, which are identical except that one is a smoker and the other is a non-smoker. Given:

i. μ_x is the force of mortality for non-smokers for $0 \le x < w$.

ii. $c\mu_x$ is the force of mortality nfor $0 \le x < \omega$, where c is a constant, $c > 1$.

Calculate the probability that the remaining lifetime of the smoker exceeds that of the non-smoker.

[11 Marks]

QUESTION A3 [*25 Marks*]

A3 (a) You are given that $q_{60} = 0.20$, $q_{61} = 0.25$, $q_{62} = 0.25$, $q_{63} = 0.30$, $q_{64} = 0.40$

i. Find ℓ_x for ages 60 to 65 begining with $\ell_{60} = 1000$

[3 Marks]

ii. Find the probability that (61) will die between the ages of 62 and 64.

[3 Marks]

iii. Given that $e_{65} = 0.8$, find e_x for $x = 60$ to 64.

[3 Marks]

(b) Given:

i. The survival function is $s(x) = 1 - x/100$ for $0 \leq x \leq 100$.

ii. The force of interest is $\delta = 0.10$.

iii. The death benefit is paid at the moment of death.

Calculate the net single premium for a 10−year endowment insurance of 50,000 for a person age $x = 50$.

[7 Marks]

(c) A 3−year term life insurance to (x) is defined by the following table

Year t	Death Benefit	q_{x+t}
0	3	0.20
1	2	0.25
2	1	0.50

Given: $v = 0.9$, the death benefits are payable at the end of the year of death and the expected present value of the death benefit is Π. Calculate the probability that the present value of the benefit payment that is actually made will exceed Π.

[9 Marks]

QUESTIONA4 [*25 Marks*]

A4 (a) Given the following information for a 3-year temporary life annuity due, contingent on the life of (x) :

Year t	Payment	p_{x+t}
0	2	0.80
1	3	0.75
2	4	0.50

and $v = 0.9$. Calculate the variance of the present value of the indicated payments.

[8 Marks]

1. Given $l_x = 100,000(100 - x)$, $0 \leq x \leq 100$ and $i = 0$.

(a) Calculate $(I\bar{a})_{95}$ exactly.

[8 Marks]

(b) Calculate the present value of a whole life annuity issued to (80). The annuity is paid continuously at an annual rate of 1 per year the first year and 2 per year thereafter.

[9 Marks]

QUESTION A5 [*25 Marks*]

A5 (a) A fully discrete last-survivor insurance of 1 is issued on two independent lives each age x . Level net annual premiums are paid until the first death. Given:

i. $A_x = 0.4$

ii. $A_{xx} = 0.55$

iii. $a_x = 9.0$

Calculate the net annual premium

[8 Marks]

(b) A whole life insurance pays a death benefit of 1 upon the second death of (x) and (y) . In addition, if (x) dies before (y), a payment of 0.5 is payable at the time of death. Mortality for each life follows the Gompertz law with a force of mortality given by $\mu_z = Be^z, z \geq 0$. Show that the net single premium for this insurance is equal to

$$\overline{A}_x + \overline{A}_y - \overline{A}_\omega(1 - 0.5c^{x-\omega})$$

where $c^\omega = c^x + c^y$.

[9 Marks]

(c) Z is the present-value random variable for an insurance on the independent lives of (x) and (y) where

$$Z = \begin{cases} v^{T(y)}, & \text{if } T(y) \leq T(x) \\ 0, & \text{otherwise} \end{cases}$$

i. (x) is subject to a constant force of mortality of 0.07.
ii. (y) is subject to a constant force of mortality of 0.09.
iii. The force of interest is a constant $\delta = 0.06$.

Calculate $Var(Z)$.

[8 Marks]

QUESTION A6 [*25 Marks*]

A6 (a) The claim made in respect of policy h is denoted S_h. The three possible values of S_h are as follows:

$$S_h = \begin{cases} 0, & \text{if the insured life (x) survives} T(y) \leq T(x) \\ 100, & \text{if the insured surrenders the policy} \\ 1000, & \text{if the insured dies} \end{cases}$$

The probability of death is

$$q_{1,x} = 0.001,$$

the probability of surrender is

$$q_{2,x} = 0.15,$$

and the probability of survival is

$$p_x = 1 - q_{1,x} - q_{2,x}.$$

Use the normal approximation to calculate the probability that the aggregate claims of five identically distributed policies

$$S = S_1 + \cdots + S_5$$

exceeds 200.

[8 Marks]

(b) Consider the compound model described by formula $S = X + \ldots + X_N$ where N, X_i are independent, and X_i are identically distributed.

i. Show that the moment generating function of S is

$$M_s(t) = M_N(log(M_x(t)))$$

where $M_N(t)$ and $M_x(t)$ are the moment generating functions of N and X.

[7 Marks]

ii. Show that $E[S] = E[N]E[X]$ and

$$E[S^2] = E[N^2]E[X]^2 + E[N](E[X^2 - E[X]^2).$$

[10 Marks]

END OF EXAMINATION PAPER