UNIVERSITY OF ESWATINI

DECEMBER 2019 MAIN EXAMINATION

BSc IV, B.Ed IV, BASS IV

Title of Paper : Numerical Analysis II

Course Number : MAT411/M411

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

A1 (a) Show that the linear least squares approximation of

$$f(x) = x^2 + 3x + 2$$

on the interval $[0, 1]$ is $P_1(x) = \frac{11}{6} + 4x$ [6 Marks]

(b) Determine if the differential equation

$$y(x)' = (2x+3)y(x) + 3, \quad 1 \leq x \leq 3, \ \ y(1) = 1$$

has a unique solution for $1 \leq x \leq 3$. [5 Marks]

(c) Use the method of undetermined coefficients to derive the 2-step Adams-Bashforth explicit method. [6 Marks]

$$y_{i+1} = y_i + \frac{h}{2}[3f(t_i, y_i) - f(t_{i-1}, y_{i-1})]$$

(d) Consider the following differential equation

$$xY'' + 2Y = 4 - 6x^2$$

which is to be approximated using finite differences.

i. Derive the finite difference scheme for the differential equation. [2 Marks]

ii. Find the local truncation error for the scheme. [5 Marks]

(e) i. Use the 2-step Adams-Moulton method given by

$$y_{i+1} = y_i + \frac{h}{12}[5f(t_{i+1}, y_{i+1}) + 8f(t_i, y_i) - f(t_{i-1}, y_{i-1})]$$

to compute the approximate solution $y(0.4)$ for the differential equation

$$y' = 2ty, \quad y(0) = 1, \quad y(0.2) = 1.040810770$$

[5 Marks]

ii. Find the error of the approximation in (i) given that the exact solution is $y(t) = e^{t^2}$. [1 Mark]

(f) Consider the heat equation $u_t = u_{xx}$

i. Derive the finite difference scheme for solving the heat equation using the forward difference in time and central difference scheme in space (FTCS). [3 Marks]

ii. Use the von-Neumann analysis analysis to prove that the BTCS scheme for solving the heat equation is conditionally stable. [7 Marks]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

B2 Use an $O(h^2)$ finite difference scheme to solve the following boundary-value problems using a step size $h = \frac{1}{4}$ and compare the results against the exact solution $U(x) = 4x^2 + 3x + 2$. [20 Marks]

$$4U''(x) - xU(x) + U(x) = 34 - 4x^2, \quad U(0) = 2, \quad U(1) = 9$$

QUESTION B3 [*20 Marks*]

B3 Solve the discretized form of the Laplace equation [20 Marks]

$$u_{xx} + u_{yy} = 1$$

using step sizes $\Delta x = 1/3$ and $\Delta y = 1/2$ for $u(x, y)$ defined in the domain $0 \leq x \leq 1$ and $0 \leq y \leq 1$ given the boundary conditions

$$u(x, 0) = -2, \quad u(x, 1) = -1, \quad u(0, y) = 0, \quad u(1, y) = 1$$

QUESTION B4 [*20 Marks*]

B4 (a) Derive the 3-step Adams-Bashforth explicit method using the Newton-Backward difference formula. [12 Marks]

(b) Compute the local truncation error for the 3-step Adams-Bashforth [8 Marks]

QUESTION B5 [*20 Marks*]

B5 (a) Find the linear polynomial that best fits the following data in the sense of least squares [10 Marks]

x	0	0.25	0.5	0.75	1
y	0	0.015625	0.125	0.421875	1

(b) Given that the first Legendre polynomial is $\phi_0(x) = 1$, use the Gram-Schmidt process to find $\phi_1(x)$ and $\phi_2(x)$ for the interval $[0, 1]$ with weight function $w(x) = 1$. [10 Marks]

QUESTION B6 [*20 Marks*]

B6 (a) Prove that the continuous least squares trigonometric polynomial $S_2(x)$ for

$$f(x) = \begin{cases} -1 & -\pi < x < 0, \\ 1 & 0 \leq x \leq \pi \end{cases}$$

is $S_2(x) = \dfrac{4 \sin x}{\pi}$ [10 Marks]

(b) Use Taylor method of order 2 to solve the IVP

$$y'(t) = 2ty, \quad y(0) = 2$$

for $0 \leq t \leq 0.4$ with $h = 0.2$ and compute the error against the exact solution $y(t) = 2e^{t^2}$. [10 Marks]

END OF EXAMINATION PAPER