UNIVERSITY OF ESWATINI 36

SUPPLEMENTARY EXAMINATION, 2019/2020

B.Ed (Pri.), (Sec.) II; B.Sc II

Title of Paper : Mathematics for Scientists

Course Number : MAT215

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

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SECTION A [40 Marks]: ANSWER ALL QUESTIONS

QUESTION A1 [40 Marks]

A1 (a) Which of the following matrices is/are not in row echelon form?

$$M = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad N = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad P = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 2 & 0 \end{pmatrix}$$

[3 marks]

(b) Find the perpendicular distance of $(-2, 3)$ from the line $4x - 3y = 8$. [3 marks]

(c) Find the centre and radius of the circle

$$x^2 + y^2 + 6x - 8y = 0.$$

[4 marks]

(d) If $\mathbf{a} = -3i + 4j + 5k$, $\mathbf{b} = 2i + 3j - 5k$ and $\mathbf{c} = -2i + 14j$. Show that $\mathbf{c}$ is parallel to $\mathbf{a} + \mathbf{b}$. [4 marks]

(e) State Rolle's Theorem. [3 marks]

(f) Find the turning point(s) of $f(x) = x^3 - 3x^2 + 2$. [3 marks]

(g) Find the area of the region bounded by the curves $y = x$ and $y = x^2 - x$. [6 marks]

(h) If $f(x, y) = x^4y^2 + xy^5 + x^3y^3$. Show that the function $f(x, y)$ is homogeneous and find the degree. [4 marks]

(i) Solve $y'' - 5y' + 6y = 0$, where $y' = \frac{dy}{dx}$. [4 marks]

(j) Let $y(t)$ be the unknown. Identify the order, degree and linearity of the following equations.

i. $(y + t)y' + y = 1$, where $y' = \frac{dy}{dt}$. [1,1,1 marks]

ii. $y''' = \cos(2ty)$. [1,1,1 marks]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [20 Marks]

B2 (a) Solve the system of linear equation

$$\begin{aligned} x - 2y + z &= -1 \\ 3x + y - 2z &= 4 \\ y - z &= 1. \end{aligned}$$

by Gauss-Jordan elimination method. [10 marks]

(b) Find the area of the quadrilateral $ABCD$ with vertices $A(2, 0)$, $B(-2, -2)$, $C(-4, -4)$ and $D(1, -7)$. [10 marks]

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QUESTION B3 [*20 Marks*]

B3 (a) State Mean Valued Theorem. [4 marks]

(b) Determine whether the function $f(x) = x^3 + x - 4$ satisfies the hypotheses of the Mean Valued Theorem on the interval $[-1, 2]$ and if so, find all c in $(-1, 2)$ such that $f(2) - f(-1) = 3f'(c)$. [7 marks]

(c) Find the Taylor Series, center at $a = 1$, for $\ln x$. [9 marks]

QUESTION B4 [*20 Marks*]

B4 (a) The monthly payment M for an instalment loan of P Emalangeni taken out t years at an annual interest rate of r (in decimal form) is

$$M = f(P, r, t) = \frac{\frac{Pr}{12}}{1 - \left(\frac{1}{1+(r/12)}\right)^{12t}}.$$

Find the monthly payment for a home mortgage of $E100,000$ taken out for 30 years at an annual interest rate of 4%. How much is the total amount paid? [12 marks]

(b) If $f(x, y) = 2x^4y^3 - xy^2 + 3y + 1$. Find

i. $f_{xy}(1, 1)$,

ii. $f_{yy}(1, 2)$. [4,4 marks]

QUESTION B5 [*20 Marks*]

B5 (a) State the classification (test) of relative extrema for functions of two variables. [7 marks]

(b) Find the relative extrema and saddle points of $f(x, y) = xy - \frac{1}{4}x^4 - \frac{1}{4}y^4$. [13 marks]

QUESTION B6 [*20 Marks*]

B6 (a) Find all solutions y of the differential equation $y' = \frac{x^2+3y^2}{2xy}$. [10 marks]

(b) Find the integrating factor and solution of

$$(1 + t^2)y' + 4ty = (1 + t^2)^{-2},\ y(0) = 1 \text{ where } y' = \frac{dy}{dt}.$$

[10 marks]

END OF EXAMINATION