University of Eswatini

Final Examination, December 2019

B.A.S.S. , B.Sc, B.Eng, B.Ed

Title of Paper : Calculus I

Course Number : MAT211

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Begin each major question (A1, B2, etc) on a new page.
3. Each question in Section B is worth 20%.
4. Show all your working.
5. Non programmable calculators may be used (unless otherwise stated).
6. Special requirements: None.
7. Indicate your program next to your student ID number.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Section A: Answer All Questions

A1.

(a) i. Consider the function $f(x) = 2x + \frac{1}{x}$. Find and classify the critical points of $f(x)$. [6]

ii. Identify the open intervals on which the function $h(x) = x^2 - 2x - 8$ is increasing or decreasing. [6]

iii. For the function $f(x) = 3x - x^2, \quad x \in [0,\ 3]$, determine whether the Rolle's Theorem can be applied to $f(x)$. If yes, find all values of c in the open interval $(a,\ b)$ such that $f'(c) = 0$. If not, explain why. [4]

iv. State L'Hopital's Rule. [3]

v. Use L'Hopital's Rule to evaluate $\lim_{x\to\infty} (\ln x)^{\frac{1}{x}}$. [5]

(b) i. The region bounded by the curves $y = \sqrt{x}$ and $y = x^2$ is rotated about the line $x = -2$ to generate a solid of revolution. Use two different methods to set up, but do not evaluate, the integral for the volume of the solid. [4]

ii. Find the area of the region enclosed by the curves, $y = x^2$, $x + y = 2$ and the x-axis. [5]

(c) i. List the first five terms of the sequence $a_n = (-1)^{(n+1)} \left(\frac{2}{n}\right)$. [3]

ii. Find the limit of the sequence $a_n = \frac{2n}{\sqrt{n^2+1}}$. [2]

iii. Find the sum of the geometric series $\sum_{n=0}^{\infty} \frac{3}{2^n}$. [2]

Section B: Answer Three(3) Questions Only

B2.

Consider the function $f(x) = \dfrac{x^2}{x-1}$.

(a) Identify the domain of $f(x)$. [1]

(b) Find and **classify** all critical points of f. [4]

(c) Find intervals where f is increasing and where it is decreasing. [4]

(d) Find possible points of inflection, if any occur and determine concavity of the graph. [3]

(e) Identify any asymptotes that may exist. [3]

(f) Sketch the graph of f labelling all major points found above including intercepts if any occur. [5]

B3.

(a) Sketch and find the area of the region bounded by the graphs of $x = y^2$ and $y = x - 6$. [10]

(b) Sketch the region bounded by the curves $y = x$, $y = x + 1$ $x = 0$, and $y = 2$. The region is rotated about the y-axis to form a solid of revolution. Carefully set up an integrand to find the volume of the solid. [10]

B4.

(a) Find the arc length of the graph of $y = \dfrac{x^3}{6} + \dfrac{1}{2x}$ over the interval $[\frac{1}{2}, 2]$. [10]

(b) The curve $y = \sqrt{4 - x^2}$, $-1 \leq x \leq 1$ is an arc of the circle $x^2 + y^2 = 4$. Find the area of the surface obtained by rotating this arc about the x-axis. [10]

B5.

(a) Use the Ratio test to show that the series $\sum_{n=1}^{\infty} \frac{n^3}{3^n}$ converges. [4]

(b) i. Express the geometric series $\sum_{n=0}^{\infty} \frac{(-4)^{3n}}{5^{n-1}}$ in the form $\sum_{n=0}^{\infty} ar^n$. [4]

ii. Analyze the convergence of the series above in i. [4]

(c) Find the 3$^{\text{rd}}$ Taylor polynomial of $f(x) = \sqrt{x}$ centered at $c = 4$. [8]

B6.

(a) Find the interval and radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{1}{(-3)^{2+n}(n^2+1)}(4x-12)^n.$$

[12]

(b) Find the Taylor series for the function $f(x) = e^{-x}$ about $x = -4$. [8]

END OF EXAMINATION