UNIVERSITY OF ESWATINI

MAIN EXAMINATION, 2019/2020

BEng I, BSc I, BEd I, BSc IT I, BSc Comp Sci ED I, BASS I, BSc IS I, BSc IT-IDE I

Title of Paper : ALGEBRA, TRIG. AND ANALYTIC GEOMETRY

Course Number : MAT111

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

A1 (a) i. State any two properties of the dot product. [2 marks]

ii. Given the vectors $\boldsymbol{A} = 3\hat{\imath} + 12\hat{\jmath} - 4\hat{k}$, $\boldsymbol{B} = 3\hat{\imath} - 8\hat{\jmath} - 5\hat{k}$ and $\boldsymbol{C} = 9\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$. Find $\boldsymbol{B}\cdot(\boldsymbol{C} + \boldsymbol{A})$ [5 marks]

(b) Find the sum of the geometric progression (G.P) $8, -4, 2, \cdots \frac{1}{128}$. [5 marks]

(c) Find the equation of a straight line that is perpendicular to the line $2y = 4 - x$ and passing throught the point $(-1, 1)$. [3 marks]

(d) Given the complex numbers $Z_1 = 2 - 2i$ and $Z_2 = 3 + 4i$

i. Express Z_1 in polar form. [3 marks]

ii. Evaluate $\frac{Z_1}{Z_2}$ and express your final anwer in the form $a + bi$. [4 marks]

(e) Given the following matrices

$$A = \begin{pmatrix} 1 & -2 \\ 4 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & -2 \\ 0 & 1 \\ 4 & -3 \end{pmatrix} \text{ and } C = \begin{pmatrix} -1 & 3 & 7 \\ 0 & 5 & -4 \\ 6 & -2 & 1 \end{pmatrix}.$$

Find (where possible)

i. AB^T. [3 marks]

ii. $\det(C)$. [5 marks]

(f) Without using a calculator, evaluate

i. $\log_3 135 - \log_3 5$. [2 marks]

ii. $(\cos 225° - \sin 255°)$. [3 marks]

(g) Using synthetic division, determine the quotient and remainder of

$$\frac{x^3 + 5x^2 + 2x - 8}{x + 2}.$$

[5 marks]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

B2 (a) Find the first four (4) terms of the binomial expansion of

$$\left(\frac{1}{x^3} + y^2\right)^{16},$$

and simplify term by term. [8 marks]

(b) A principal sum of money M Emalangeni is invested in an account that pays interest at $r\%$ p.a. compounded daily. After t years, the principal amount in the account is given by

$$A(t) = M\left(1 + \frac{r}{365}\right)^{365t}.$$

i. If the principal amount of the account triples in 8 years, find r. [7 marks]

ii. Find the time it takes for the principal amount on the account to double. [5 marks]

QUESTION B3 [*20 Marks*]

B3 (a) Find all the roots of $2x^3 + x^2 - 13x + 6 = 0$. [6 marks]

(b) Solve for x in the equations:

i. $4 \times 5^{x+3} = 7^{2-x}$. [5 marks]

ii. $\log_2 x + \log_2(x-1) = 1$. [3 marks]

(c) The sum of the first 17 terms of an arithemtic progression (A.P) is zero. If the 6^{th} term is 21, determine the first three terms. [6 marks]

QUESTION B4 [*20 Marks*]

B4 (a) Given the equation: $2\cos^2 x + 3\cos x + 1 = 0$.

i. Find the general solution of the equation. [6 marks]

ii. Find the particular solution, in radians, in the interval $0 < x \leq 2\pi$. [3 marks]

(b) Prove the trigonometric identity

$$\frac{2\cos^2\theta}{2\cot\theta - \sin 2\theta} = \tan\theta.$$

[6 marks]

(c) Find the angle between the vectors $\boldsymbol{A} = 3\hat{\imath} + 12\hat{\jmath} - 4\hat{k}$ and $\boldsymbol{B} = 3\hat{\imath} - 8\hat{\jmath} - 5\hat{k}$. [5 marks]

QUESTION B5 [*20 Marks*]

B5 (a) Solve the following system of linear equations using Crammer's rule

$$\begin{aligned} x + 2y + z &= 1 \\ x - y - z &= 0 \\ 2x + y + z &= 3 \end{aligned}$$

[15 marks]

(b) FInd the radius and centre of the circle

$$x^2 + y^2 - 6x + 8y - 11 = 0.$$

[5 marks]

QUESTION B6 [*20 Marks*]

B6 (a) Prove using the method of mathematical induction that: $P(n) = 7^n - 2^n$ is always divisible by 5 for integer values of $n \geq 1$. [6 marks]

(b) Calculate the value of the sum

$$\sum_{k=0}^{\infty} 50 \left(\frac{4}{9}\right)^k.$$

[4 marks]

(c) Use de Moivre's theorem to find the square root of $2 - 2\sqrt{3}i$. [6 marks]

(d) Find the 10^{th} term of the binomial expansion

$$\left(x^4 + \frac{y^2}{x^2}\right)^{18}.$$

[4 marks]

END OF EXAMINATION