UNIVERSITY OF ESWATINI

EXAMINATION, 2018/2019

BASS, B.Ed (Sec.), B.Sc.

Title of Paper : Foundations of Mathematics

Course Number : MAT231/M231

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SEVEN (7) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, A2, B2, ..., B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*20 Marks*]

(a) Determine whether or not the given sentence is a proposition. If it is a proposition, give its truth value. (6)

i. Lend me your pen.
ii. The moon is a sphere.
iii. n is an integer.
iv. $\forall x \in \mathbb{R}, x^2 < 2$.

(b) Give clear definitions of each of the following

i. A *relation* from a set A into a set B? (2)
ii. A *function* from a set A into a set B. (2)
iii. A *partial order* on a set A. (4)

(c) Write down (i.) the inverse, (ii.) the converse, and (iii.) the contrapositive of the following statement.

"If I cannot find accommodation on campus, then I have to live at home." (6)

QUESTION A2 [*20 Marks*]

(a) In each of the following arguments, state (do not prove) whether it is valid or not. If it is valid, state whether it is modus ponens or modus tollens. If it is not valid, state whether it is the inverse error or converse error. (8)

i.	ii.	iii.	iv.
$p \to q$	$p \to q$	$p \to q$	$p \to q$
$\neg p$	$\neg q$	q	p
$\therefore \neg q$.	$\therefore \neg p$.	$\therefore p$.	$\therefore q$.

(b) Use a truth table to determine whether or not the following argument is valid. (6)

$$\begin{array}{rc} & (p \wedge q) \to r \\ & \neg p \vee \neg q \\ \hline \therefore & \neg r \end{array}$$

(c) Without using truth tables, show that $\neg[(p \to \neg q) \vee \neg(r \wedge \neg r)] \equiv c$. (6)

END OF SECTION A – TURN OVER

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B3 [20 *Marks*]

(a) Prove: *The product of two odd integers is odd.* (5)

(b) Prove: *For an integer n, if $5n$ is even, then n is even.* (5)

(c) Prove: *For any integer n, the number $n^3 - n$ is even.* (5)

(d) Let $a, b, c \in \mathbb{Z}, a \neq 0, b \neq 0$. Prove: *If $a \mid b$ and $b \mid c$, then $a \mid c$.* (5)

QUESTION B4 [20 *Marks*]

(a) Let $A = \{1,2,3\}, B = \{3,4\}$ be sets in the universal set $U = \{n \in \mathbb{Z} : 0 \leq n \leq 5\}$. Write down the following sets. (6)

i. $A \cap B$
ii. $A \cup B$
iii. $A \setminus B$
iv. $(A \cap B)^c$
v. $B \times A$
vi. $\mathscr{P}(B)$

(b) Let A and B be sets in a universal set U. Prove

i. $\emptyset \subseteq A$. (4)
ii. If $A \subseteq B$, then $A \cap B = A$. (4)
ii. $(A \cap B)^c = A^c \cup B^c$. (6)

QUESTION B5 [20 *Marks*]

(a) Use mathematical induction to prove that $3^{2n} + 7$ is divisible by 8 for all integers $n \geq 0$. (7)

(b) Use strong induction to prove: *Any integer $n > 1$ is either a prime number or can be written as a product of prime numbers.* (7)

(c) Find a solution to the sequence recursively defined by

$$a_1 = 1,\ a_2 = 4, \quad a_n = 2a_{n-1} - a_{n-2},\ n \geq 3.$$

(6)

QUESTION B6 [20 *Marks*]

(a) Let $A = \{a, b, c, d\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\}$. Which of the following relations from A into B are functions? Explain your answer. (4)

i. $\{(a, 4), (d, 3), (b, 3), (c, 2)\}$ ii. $\{a, 5), (c, 4), (d, 3)\}$.

(b) Let $f : A \to B$ be a function. What does it mean to say that

i. f is an *injection*, ii. f is a *surjection*?

(4)

(c) Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(n) = 2n - 3$.

i. Show that the range of f is the set of odd integers. (2)

ii. Is f surjective? Explain. (2)

iii. Show that f is injective. (4)

(d) Let $f : A \to B$ and $g : B \to C$ be surjective functions. Show that $g \circ f : A \to C$ is also surjective. (4)

QUESTION B7 [20 *Marks*]

(a) Let R be a relation on a set A. Explain what it means to say

i. R is *reflexive*, iii. R is *symmetric*,

ii. R is *anti-symmetric*, iv. R is *transitive*.

(4)

(b) Let $A = \{1, 2, 3, 4\}$. Given that

$$R = \{(1, 1), (1, 3), (2, 2), (2, 4), (3, 1), (3, 3), (4, 2), (4, 4)\}$$

is an equivalence relation on A (no need to prove this), write down the equivalence classes of R. (4)

(c) Define the relation $\sim$ on $\mathbb{Z}$ as follows: For $m, n \in \mathbb{Z}$, $m \sim n$ if and only if $m - n$ is divisible by 3. Show that $\sim$ is an equivalence relation on $\mathbb{Z}$. (6)

(d) Let $\mathscr{A}$ be a collection of sets. Let R be the relation on $\mathscr{A}$ defined as follows: For $A, B \in \mathscr{A}$, $(A, B) \in R$ if and only if $A \subseteq B$. Show that R is a partial order on $\mathscr{A}$. (6)

END OF EXAMINATION PAPER