UNIVERSITY OF ESWATINI

MAIN EXAMINATION, 2018/2019

B.Ed (Pri.), (Sec.) II; B.Sc II

Title of Paper : Mathematics for Scientists

Course Number : MAT215

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

A1 (a) Calculate the value(s) of x for which $A(-2,1)$ and $B(x,-7)$ are equidistant from $P(1,-4)$. [3 marks]

(b) Find the centre and radius of the circle

$$2x^2+2y^2+x-11y=0.$$

[4 marks]

(c) If $\vec{x}=2\hat{i}+\hat{k}$ and $\vec{y}=(4,1,-2)$, find $\vec{x}\times\vec{y}$. [3 marks]

(d) Let $A=\begin{pmatrix} x+2 & -3 \\ 0 & x-3 \end{pmatrix}$, find the values of x if A is a singular matrix. [3 marks]

(e) State Rolle's Theorem. [2 marks]

(f) Evaluate $\lim_{x\to 2}\frac{x^2-4}{x-2}$. [3 marks]

(g) Find the critical point(s) of $f(x,y)=x^3+y^2-3x-y$. [4 marks]

(h) State two properties of double integrals over a given region D. [4 marks]

(i) State Fubini's Theorem for triple integral. [4 marks]

(j) Classify each of the following differential equations by stating the order and degree.

i. $(y''')^4+x^2y''-2y(y')^6+xy=0$,

ii. $ay''+by'+c=0$,

iii. $y^{(iv)}+(y')^4=x^2$. [2,2,2 marks]

(k) Solve $y''-4y'=0$. [4 marks]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

B2 (a) Let $B = \begin{pmatrix} 0 & 2 & 0 \\ 1 & 0 & 4 \\ 0 & -2 & 1 \end{pmatrix}$. Find

i. all the minors,
ii. all the cofactors,
iii. Adjoint matrix,
iv. Inverse matrix. [5,2,2,3 marks]

(b) Solve the system of linear equation

$$\begin{aligned} x + y + z &= 5 \\ 2x + 3y + 5z &= 8 \\ 4x + 5z &= 2. \end{aligned}$$

by Gauss-Jordan elimination method. [8 marks]

QUESTION B3 [*20 Marks*]

B3 (a) Determine whether the function $f(x) = x^3 - 8x - 5$ satisfies the hypotheses of the Mean Valued Theorem on the interval $[1, 4]$ and if so, find all c in $(1, 4)$ such that $f(4) - f(1) = 3f'(c)$. [6 marks]

(b) Apply L'Hôpital rule to evaluate $\lim_{x \to 0} \frac{e^{2x}-1}{x}$. [4 marks]

(c) Find the first four terms of the Taylor series expansion of $\sin x$ about $x = 0$. [4 marks]
Use this series to approximate to four decimal places $\int_0^1 \sin x^2 dx$. [6 marks]

QUESTION B4 [*20 Marks*]

B4 (a) If $f(x, y) = 2x^3y^2 + 2y + 4x$, find

i. $f_x(1, 2)$,
ii. $f_y(1, 2)$. [4,4 marks]

(b) If $f(x, y) = x^2y^3 + x^4y$, show that $f_{xy} = f_{yx}$. [4 marks]

(c) If $z = 2x^2 + 3xy - 4y^2$, where $x = \cos t$ and $y = \sin t$, find $\frac{dz}{dt}$, and evaluate $\frac{dz}{dt}$ at $t = \pi/2$. [8 marks]

QUESTION B5 [*20 Marks*]

B5 (a) Find the relative maxima and minima of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. [9 marks]

(b) Apply Lagrange's method to obtain the maximum value of the function $f(x, y, z) = xy + z^3$, if x, y, z satisfy $x + y + z = 1$. [11 marks]

QUESTION B6 [*20 Marks*]

B6 (a) Evaluate $\iint_R y^2 x dA$ over the rectangle $R = (x, y) : -3 \leq x \leq 2,\ 0 \leq y \leq 1$. [10 marks]

(b) Solve the differential equation $(y^2 - 1)dx - 2(2y + xy)dy = 0$, [10 marks]

END OF EXAMINATION