UNIVERSITY OF ESWATINI

JULY 2019 RE-SIT EXAMINATION

B.Com II,III,IV

Title of Paper : Quantitative Techniques

Course Number : MAT202

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

A1 (a) Use the second derivative test to show that the function

$$f(x,y) = -4x^2 - 4xy + 180x + 2y^2 + 12y - 100$$

has a saddle point at the critical point $(x^*, y^*) = (16, 13)$ [5]

(b) A certain country's production is described by the function

$$f(x,y) = 10200x^{\frac{1}{3}}y^{\frac{2}{3}}$$

units, when x units of labour and y units of capital were used. What is the marginal productivity of labour and the marginal productivity of capital when the amounts spent on labour and capital are 64 units and 125 units, respectively? [6]

(c) The first two steps for the Simplex method solution of the following linear programming problem

$$\begin{aligned} \text{Maximize} \quad P &= 5x_1 + 3x_2 \\ \text{Subject to} \quad 2x_1 + x_2 &\leq 6 \\ x_1 + x_2 &\leq 5 \\ x_1, x_2 &\geq 0 \end{aligned}$$

are given in the table below.

STEP 1

Basic	x_1	x_2	x_3	x_4	RHS
x_3	2	1	1	0	6
x_4	1	1	0	1	5
Profit	-5	-3	0	0	0

STEP 2

Basic	x_1	x_2	x_3	x_4	RHS
x_1	1	$\frac{1}{2}$	$\frac{1}{2}$	0	3
x_4	0	$\frac{1}{2}$	$-\frac{1}{2}$	1	2
Profit	0	$-\frac{1}{2}$	$\frac{5}{2}$	0	15

where x_3 and x_4 are slack variables. Do the remaining step(s) of the Simplex method to obtain the optimal solution. [7]

(d) Consider the following matrix

$$A = \begin{bmatrix} -2 & 3 & 1 \\ 2 & -1 & 4 \\ 1 & -1 & 1 \end{bmatrix}$$

By applying several row operations of the Gauss-Jordan procedure, the system can be converted to the following augmented form

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & -2 \\ 0 & 1 & 3 & 1 & 0 & 2 \end{array}\right]$$

Starting from this augmented form, complete the rest of the Gauss-Jordan method steps to find the inverse of A. [6]

(e) As a savings program toward Alfred's university education, his parents decide to deposit E100 at the end of every month into a bank account paying interest at the rate of 6% per year compounded monthly. If the savings program began when Alfred was 6 years old, how much money would have accumulated by the time he turns 18? [4]

(f) A company will need to replace a piece of equipment at a cost of E900,000 in 10 years. To have this money available in 10 years' time, a sinking fund is established by making equal monthly deposits into an account paying 4% compounded monthly. How much should each payment be? [4]

(g) Find the best transportation schedule for the data given in the table below using the North-West corner rule and the Stepping Stone method.

From\To	1	2	3	Supply
A	5	4	3	470
B	2	8	1	130
Demand	20	340	240	600

[8]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

B2 (a) The profits from the sale of x units of batteries for Samsung cellphones and y units of batteries for Huawei cellphones is given by

$$P(x, y) = -x^2 + 28x - y^2 + 8y + 2000$$

Use the method of Lagrange multipliers to find the values of x and y that lead to the maximum profit if the firm must produce 48 units of batteries. Find the maximum profit. [8]

(b) Suppose x units of labour and y units of capital are required to produce

$$f(x, y) = 3100\sqrt{x}\sqrt{y}$$

units of a certain product. Each unit of labour costs E6 and each unit of capital costs E4. If E192 has been budgeted for the production of this product, how should this amount be allocated between labour and capital in order to maximize production? What is the maximum number of units that can be produced? [12]

QUESTION B3 [*20 Marks*]

B3 Curios Swaziland produces two types of souvenirs: Emagcebesha and Buhlalu wrist braces. Selling ligcebesha yields a profit of E3, and selling each wrist brace will result in a profit of E5. To make ligcebesha requires 1 minute on machine I and 1 minute on machine II. A wrist brace requires 2 minutes on machine I and 1 minute on machine II. There are 8 hours available on machine I and 7 hours available on machine II.

(a) Formulate the problem as a linear programming problem. [5]

(b) Use the Simplex method to find the number of each type of souvenir that Curios Swaziland should make in order to maximize its profit? [15]

QUESTION B4 [*20 Marks*]

B4 (a) Use the Adjoint method to compute the inverse of the matrix

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

[10]

(b) Let

$$A = \begin{bmatrix} 0.4 & 0.2 \\ 0.3 & 0.1 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 10 \\ 14 \end{bmatrix}.$$

Solve the matrix equation $X - AX = D$ for X. [10]

QUESTION B5 [*20 Marks*]

B5 A construction company has four tractors located at four different garages. The tractors are to be moved to four different farming sites. The distances in kilometres between the tractors and the farming sites are given in the table below:

	Tractors			
Farms	Tractor 1	Tractor 2	Tractor 3	Tractor 4
Farm 1	58	56	35	43
Farm 2	58	46	10	28
Farm 3	33	24	55	14
Farm 4	34	12	16	16

How should the tractors be moved to the farming sites in order to minimize the total distance travelled? [20]

QUESTION B6 [*20 Marks*]

B6 (a) Find and classify all the extreme values of the function

$$f(x, y) = x^3 + 3x^2 - 12y^3 - 18y^2 - 10$$

[9]

(b) A dietician wants to design a breakfast menu for certain hospital patients. The menu is to include two items A and B. Suppose that each Kilogram of A provides 2 units of vitamin C and 2 units of iron and each Kilogram of B provides 1 unit of vitamin C and 2 units of iron. Suppose the cost of A is 4 Emalangeni per Kg and the cost of B is 3 Emalangeni per Kg. The breakfast menu must provide at least 8 units of vitamin C and 10 units of iron.

i. Formulate the problem as a linear programming problem. [3]

ii. Use the Graphical method to determine the number of Kilograms of each item that should be provided in order to meet the iron and vitamin C requirements for the least cost. [8]

END OF EXAMINATION PAPER