UNIVERSITY OF ESWATINI

JUNE 2019 MAIN EXAMINATION

B.Com II,III,IV

Title of Paper : Quantitative Techniques

Course Number : MAT202

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

A1 (a) The profit function arising from producing and selling x units of product A and y units of product B is given by

$$P(x,y) = -9x^2 - 8xy + 1580x - 3y^2 + 800y - 20000$$

It can easily be shown that $x = 70$ and $y = 40$ is the only critical point for the profit function P.

i. Use the second derivative test to show that $(70, 40)$ gives maximum profit. [5]

ii. Compute the maximum profit. [1]

(b) Find a classify all the critical points of

$$f(x,y) = x^2 - 6xy + 108x + y^2 - 4y - 10$$

[6]

(c) A manufacturing company has determined that the demand equations for selling two of its products are modelled as

$$p = 6 - 2x - y$$
$$q = 6 - x - 2y$$

where x and y denote the number of units of the products sold in one week. Find the total revenue function $R(x,y)$ and sketch the corresponding domain. [6]

(d) Consider the following LP model for determining the levels of production of two products, A and B:

$$\begin{aligned} \text{Maximize} \quad & P = 4x_1 + 3x_2 \\ \text{Subject to} \quad & 2x_1 + x_2 \le 800 \quad \text{(Resource 1)} \\ & x_1 + x_2 \le 600 \quad \text{(Resource 2)} \\ & x_1, x_2 \ge 0 \end{aligned}$$

where x_1 and x_2 are the units of products A and B, respectively, to be produced. The solution of the LP by the Simplex method proceeds as follows:

	Basic	x_1	x_2	x_3	x_4	RHS
STEP 1	x_3	2	1	1	0	800
	x_4	1	1	0	1	600
	P	-4	-3	0	0	0

	Basic	x_1	x_2	x_3	x_4	RHS
STEP 2	x_1	1	$\frac{1}{2}$	$\frac{1}{2}$	0	400
	x_4	0	$\frac{1}{2}$	$-\frac{1}{2}$	1	200
	P	0	-1	2	0	1600

where x_3 and x_4 are slack variables. Do the remaining step(s) of the Simplex method to obtain the optimal solution. [5]

(e) Consider the following linear system of equations

$$\begin{aligned} x_1 - 2x_2 + 8x_3 &= 9 \\ -2x_1 + 2x_2 + x_3 &= 3, \\ x_1 + 2x_2 - 3x_3 &= 8 \end{aligned}$$

By applying several row operations of the Gauss-Jordan procedure, the system can be converted to the following augmented form

$$\left[\begin{array}{ccc|c} 1 & 0 & 9 & 12 \\ 0 & 1 & -6 & -2 \\ 0 & 2 & 19 & 27 \end{array}\right]$$

Starting from this augmented form, complete the rest of the Gauss-Jordan method steps to find x_1, x_2 and x_3. [5]

(f) Find the amount of an ordinary annuity consisting of 12 monthly payments of E200 that earn interest at 12% per year compounded monthly. [4]

(g) Find the present value of an ordinary annuity consisting of 36 monthly payments of E200 each and earning interest at 8% per year compounded monthly. [4]

(h) Find the initial solution of the transportation problem given by the data in the table below.

From\To	1	2	3	Supply
A	1	11	8	40
B	7	10	6	50
C	9	12	4	310
Demand	10	330	60	400

[4]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

B2 (a) A certain country's production in the early years following World War II is described by the function

$$f(x, y) = 6000x^{\frac{1}{5}}y^{\frac{4}{5}}$$

units, when x units of labour and y units of capital were used.

i. Compute f_x and f_y. [4]

ii. What is the marginal productivity of labour and the marginal productivity of capital when the amounts expended on labour and capital are 3125 units and 7776 units, respectively? [4]

iii. Should the government have encouraged capital investment rather than increasing expenditure on labour to increase the country's productivity? [2]

(b) Suppose that a consumer has the utility function

$$f(x, y) = x^2y$$

where x is the number of units of commodity A and y is the number of units of commodity B. Suppose further that the consumer has E54 to spend. If a unit of the commodity A costs E6 and a unit of B costs E1, use the method of Lagrange multipliers to maximise the utility. [10]

QUESTION B3 [*20 Marks*]

B3 (a) An agricultural research centre recommends that farmers must use two types of fertilizer: phosphate and nitrogen. It is recommended that farmers must spread out at least 8000 kg of phosphate fertilizer and not less than 6000 kg of nitrogen fertilizer to raise the productivity of their crops on farms. There are two mixtures A and B from which these fertilizers can be obtained. The costs of mixture A and B are E40 and E30, respectively. Mixture A contains 20 kg of phosphate and 10 kg of nitrogen while the Mixture B contains 10 kg of phosphate and 10 kg of nitrogen.

i. Formulate the problem as a linear programming problem (LPP). [6 Marks]

ii. Use the graphical method to solve the LPP to find the amounts of bags of each type of fertilizer that a farmer should buy to get the desired amount of fertilizers at the minimum cost

[14 Marks]

QUESTION B4 [*20 Marks*]

B4 (a) A company has two interacting branches, A and B. Branch A consumes $E0.4$ of its own output and $E0.3$ of B's output for every E1 it produces. Branch B consumes $E0.2$ of A's output and $E0.5$ of its own output per $E1$ of output. Suppose that branch A produces E50,000 worth of its product and branch B outputs E40,000 worth of its product.

i. Deduce the technology matrix and demand vector associated with the given information. [2]

ii. How much each branch should produce per month in order to meet exactly a monthly external demand of $E50,000$ for (A)- product and $E40,000$ for (B)-product? [8]

(b) Use the Gauss-Jordan method to compute the inverse of the matrix

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

[10]

QUESTION B5 [*20 Marks*]

B5 (a) Find the best transportation schedule for the data given in the table below using the North-West corner rule and the Stepping Stone method.

From\To	1	2	3	4	Supply
A	4	9	7	11	310
B	12	3	14	13	270
Demand	60	380	30	110	580

[12]

(b) A department has four employees with four jobs to be performed. The time (in hours) each men will take to perform each job is given in the effectiveness matrix.

	Jobs			
Employee	**Job 1**	**Job 2**	**Job 3**	**Job 4**
E1	52	34	46	31
E2	38	25	34	36
E3	58	54	14	59
E4	47	27	38	21

How should the jobs be allocated, one per employee, so as to minimize the total man-hours? [8]

QUESTION B6 [*20 Marks*]

B6 (a) Use the Simplex method to solve the following linear programming problem [12]

$$\begin{aligned} \text{Minimize} \quad C &= 4x_1 + 3x_2 \\ \text{Subject to} \quad x_1 + x_2 &\geq 3 \\ 2x_1 + x_2 &\geq 4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(b) i. As a savings program toward Alfred's university education, his parents decide to deposit E100 at the end of every month into a bank account paying interest at the rate of 6% per year compounded monthly. If the savings program began when Alfred was 6 years old, how much money would have accumulated by the time he turns 18? [4]

ii. A company will need to replace a piece of equipment at a cost of E900,000 in 10 years. To have this money available in 10 years' time, a sinking fund is established by making equal monthly deposits into an account paying 4% compounded monthly. How much should each payment be? [4]

END OF EXAMINATION PAPER