UNIVERSITY OF ESWATINI

Supplementary/Resit Examination, July 2019

B.A.S.S., B.Comm, B. Ed, D.Comm(IDE)

Title of Paper : Calculus for Business Studies

Course Code : MAT108/MS102

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
   Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
   Answer ANY THREE questions.
   Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Special requirements: None

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

a) Evaluate the following limits

i) $\lim_{x \to 1} \frac{2x^2 - 2x}{x - 1}$. [2]

ii) $\lim_{x \to 0} \frac{x^3 - 2x^2}{2x^4 + 6x^2}$. [3]

b) Find the first derivative of [5]

$$y = 4\sqrt{x} + 2e^x + 4\sin(x) - 7\log(x) + 5.$$

c) It was observed that the revenue after selling four quantities is E120.00. Determine the revenue function given that the marginal revenue is [5]

$$R'(x) = 2x(x^2 - 3)^{10} - \sqrt{x} + 4.$$

d) Integrate [5]

$$\int \left(\frac{7}{x} + 3x(x^3 - 2)^9 + 4e^x - 5\cos(x) \right) dx$$

e) Find the exact cost of producing the 24th bike if the cost of producing x bikes is [5]

$$C(x) = 900 + 80x + 0.01x^2$$

f) Determine the relative maxima of [5]

$$f(x) = x^3 + 3x^2 - 9x + 5.$$

g) Find the area bounded by $y = x^2$ and $y = 8 - x^2$. [5]

h) Consider the demand function [5]

$$D(x) = 400 - 30x^2$$

and the supply function

$$S(x) = 10x^2 + 120x.$$

Find the consumer's surplus at the equilibrium price level.

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

a) Use the limit definition of the derivative to find the derivative, $p'(x)$ of the function [5]

$$p(x) = x^2 - 3.$$

b) Evaluate the following limits

i) $\lim_{x \to -4} \frac{x+4}{x^2 + 6x + 24}$ [5]

ii) $\lim_{x \to \infty} \frac{3x^3 + 9x - 5}{4x^3 - 12x + x^2}$ [5]

c) Consider the function defined by

$$f(x) = \begin{cases} 2x^2 + 4x, & \text{if } x \geq 1 \\ k - x, & \text{if } x < 1 \end{cases}.$$

Determine the value of k that will make the function continuous. [5]

QUESTION B3 [*20 Marks*]

a) The total profit (in Emalangeni) from the sale of x cars is $P(x) = 20x - 0.02x^2 - 320$.

i) Find the average profit per watch if 19 cars are produced. [4]

ii) Find the marginal average profit at a production level of 19 cars and interpret. [6]

b) Use logarithmic differentiation to find the derivative, y' given $y = (4x^2 - 8)^{\ln x}$. [5]

c) Find the fourth derivative of the $g(t) = 4t^3 - 3t\sin(2t) - 3$. [5]

QUESTION B4 [*20 Marks*]

Consider the function $y = x^3 - 6x^2$.

a) Find all critical values. [3]

b) Find intervals of increase and decrease. [6]

c) Find all possible inflection points. [2]

d) Find the intervals where the curve is concave up and concave down. [4]

e) Sketch the curve showing clearly, all points of inflection, relative maximum or minimum, $y-$ intercepts and $x-$ intercepts where applicable. [5]

QUESTION B5 [*20 Marks*]

a) A company manufactures and sells x transistors per week. If the weekly cost is

$$C(x) = 5000 + 2x$$

and the weekly demand is

$$p = 10 - \frac{x}{1000},$$

where p is the price per unit in Emalangeni.

i) Determine the profit. [3]

ii) What price per transistor must be charged in order to make maximum profit? [7]

b) The rate of growth of the population $P(t)$ of a new city t years after its incorporation is estimated to be [10]

$$\frac{dP}{dt} = 400 + 600\sqrt{t}.$$

If the population was 5000 at the time of incorporation, find the population 15 years later.

QUESTION B6 [*20 Marks*]

a) Consider the demand function $D(x) = 60 - \frac{1}{x^2}$ and the supply function $S(x) = \frac{x^2}{5} + 30$. The equilibrium quantity is $x^* = 10$ and the equilibrium price is $p^* = 50$.

i) Find the consumer's surplus at the equilibrium price level. [5]

ii) Find the producer's surplus at the equilibrium price level. [5]

b) Evaluate the following integrals;

i) $\int 2x\sin(x)dx.$ [5]

ii) $\int \ln(2x+1)dx.$ [5]