University of Swaziland

Final Examination, May 2018

B.Sc II, B.A.S.S II, B.Ed II, B.Eng II

Title of Paper : Calculus II

Course Code : MAT212/M212

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.

   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.

   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.

2. Each question in Section B is worth 20%.

3. Show all your working.

4. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) (i) Sketch the curve by using the parametric equations $x = 1-t^2, y = t-2, -2 \leq t \leq 2$ to plot points. Indicate with an arrow the direction in which the curve is traced as t increases and also eliminate the parameter to find a Cartesian equation of the curve. [5]

(ii) Find an equation of the tangent to the curve $x = 1 + \ln t, y = t^2 + 2$, at the point $(1, 3)$ by two methods: (a) without eliminating the parameter and (b) by first eliminating the parameter. [5]

(iii) Plot the point whose polar coordinates are given by $(2, -2\pi/3)$. Then find the Cartesian coordinates of the point. [5]

(b) (i) Sketch the graph of the function $f(x, y) = 6 - 3x - 2y$ [4]

(ii) Verify that the function $U = e^{-\alpha^2 k^2 t} \sin kx$ is a solution of the heat conduction equation $U_t = \alpha^2 U_{xx}$ [4]

(iii) Find the directional derivative of the function $f(x, y) = e^x \sin y$ at $(0, \pi/3)$ in the direction of the vector $\mathbf{v} =< -6, 8 >$. [5]

(c) (i) Evaluate the double integral $\int\int_R (x - 3y^2)dA$ where

$$R = \{(x, y) : 0 \leq x \leq 2,\ 1 \leq y \leq 2\}.$$ [6]

(ii) Evaluate the iterated integral $\int_{-3}^{3} \int_0^{\sqrt{9-x^2}} \sin(x^2 + y^2)dydx$ by converting to polar coordinates. [6]

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

(a) What curve is represented by the polar equation $r = 2$? [4]

(b) Find the area under one arch of the cycloid

$$x = r(\theta - \sin\theta),\ y = r(1 - \cos\theta).$$ [8]

(c) Find the length of one arch of the cycloid

$$x = r(\theta - \sin\theta),\ y = r(1 - \cos\theta).$$ [8]

Question 3

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ of $x = e^t, y = te^{-t}$. For which values of t is the curve concave upward? [10]

(b) Sketch the curve $r = 1 + \sin\theta$. [10]

Question 4

(a) Find and sketch the domain of the function $f(x,y) = \frac{\sqrt{x+y+1}}{x-1}$. [6]

(b) Find the limit, if it exists, or show that the limit does not exist for

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}.$$ [6]

(c) Find the local maximum and minimum values and saddle point(s), if any, of the function

$$f(x,y) = x^2 + xy + y^2 + y.$$ [8]

Question 5

(a) Show that $f(x, y) = xe^{xy}$ is differentiable at $(1, 0)$ and find its linearization there. Then use it to approximate $f(1.1, -0.1)$. [10]

(b) Use the chain rule to find $\frac{\partial z}{\partial s}, \frac{\partial z}{\partial t}, \frac{\partial z}{\partial u}$ of
$z = x^4 + x^2y, \quad x = s + 2t - u, \quad y = stu^2.$ [10]

Question 6

(a) Find the mass and center of mass of a triangular lamina with vertices $(0, 0), (1, 0)$ and $(0, 2)$ if the density function is $\rho(x, y) = 1 + 3x + y$. [10]

(b) Evaluate $\int\int\int_B e^{(x^2+y^2+z^2)^{3/2}} dV$, where B is a unit ball:
$B = x^2 + y^2 + z^2 \leq 1$. [10]

End of Examination Paper