University of Swaziland

Final Examination, December 2017

B.Sc II, B.A.S.S II, B.Ed II, B.Eng II

Title of Paper : Calculus I

Course Code : MAT211/M211

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Each question in Section B is worth 20%.
3. Show all your working.
4. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A: ANSWER ALL QUESTIONS

Question 1

(a) (i) State Rolle's Theorem. [3]

(ii) State Extreme Value Theorem. [2]

(iii) True or False. If $f'(c) = 0$, then f has a local maximum or a local minimum at $x = c$. Explain your answer. [2]

(iv) Explain each term: (a) relative maximum and (b) absolute maximum. [2]

(b) (i) State the Mean Value Theorem for Integrals. [3]

(ii) Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on the sides by $x = 0$ and $x = 1$. [4]

(c) (i) State the Root Test for series convergence. [3]

(ii) State the Ratio Test for series convergence. [3]

(iii) True or false? If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ is convergent. Explain your answer. [3]

(iv) Suppose $a_1 + a_2 + a_3 + a_4 + \cdots = \sum_{n=1}^{\infty} a_n$ converges to l, where l is a real number. If a_3, a_5, and a_7 are removed from the series what can you say about the convergence of the series? Explain your answer. [3]

(vi) State the alternating series convergence test (Leibitz Test on Alternating Series). [3]

(v) Show that the sequence $< \dfrac{n}{n+1} >$ is monotonically increasing. [4]

(vii) Determine the radius of convergence and interval of convergence for the

SECTION B: ANSWER ANY 3 QUESTIONS

Question 2

(a) Verify that the function $f(x) = \sqrt{x^2 - 4}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[2, 4]$. [6]

(b) Verify Rolle's Theorem for the function $\frac{\sin x}{e^x}$ in the interval $[0, \pi]$. [6]

(c) A wire of given length is cut into two portions which are divided into the shapes of a circle and a square respectively. Show that the sum of the areas of the circle and the square will be least when the side of the square is equal to the diameter of the circle. [8]

Question 3

(a) Evaluate the following limits.

(i) $\lim_{x \to 0} \frac{e^{ax} - e^{-ax}}{\ln(1 + bx)}$ [4]

(ii) $\lim_{x \to 0} \left[\frac{1}{e^x - 1} - \frac{1}{x} \right]$ [4]

(iii) $\lim_{x \to 0} (\cos x)^{\cot^2 x}$ [4]

(b) Sketch the graph of the function $y = x^2 e^x$. (Show the necessary details) [8]

Question 4

(a) Find the average value of the function $f(x) = \sqrt[3]{x}$ on the interval $[1, 8]$. [6]

(b) Determine the length of $y = \ln(\sec x)$ between $0 \le x \le \pi/3$. [6]

(c) Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1. [8]

Question 5

(a) Find a formula for the general term a_n of the sequence $\frac{2}{1^2+1} + \frac{2^2}{2^2+1} + \frac{2^3}{3^2+1} + \frac{2^4}{4^2+1} + \cdots$ [4]

(b) Prove that the sequence $a_n = \sqrt{n+1} - \sqrt{n}$ is bounded. [8]

(c) Examine the series converges or diverges

$$\sum_{n=1}^{\infty} \frac{n^3}{3^n}.$$

[8]

Question 6

(a) Using comparison test determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$. [4]

(b) Determine whether the series $\sum_{n=1}^{\infty} \frac{(-10)^n}{n!}$ is absolutely convergent, conditionally convergent, or divergent. [6]

(c) Find the Taylor series for $f(x) = x^{-2}$, centered at $c = 1$. [10]

End of Examination Paper