University of Swaziland

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Final Examination December, 2016

B.Comm (IDE), D.Comm (IDE), B.Ed (IDE)

Title of Paper : Quantitative Techniques

Course Number : MS202

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of TWO sections.
   a. **SECTION A(COMPULSORY): 40 MARKS**
      Answer ALL QUESTIONS.
   b. **SECTION B: 60 MARKS**
      Answer ANY THREE questions.
      Submit solutions to ONLY THREE questions in Section B.
2. Show all your working.
3. Start each question on a fresh page.
4. Non programmable calculators may be used (unless otherwise stated).
5. Special requirements: None.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A

Answer ALL questions from section A.

A1. (a) A shop sells x kg of sugar and y kg of salt in a month at prices p Emalangeni per kg and q Emalangeni per kg respectively. The demand equations for the company are

$$x = 300 + p, \quad y = 200 + q,$$

and its cost function is $1000 - 3xy$.

i. Determine the monthly revenue function $R(x, y)$ for the company. [4]

ii. Evaluate $R_y(40, 10)$ and interpret your results. [2]

(b) A factory produces x desks and y chairs. Let $C = 12x^2 + 24y^2$ be the joint cost of production of x and y. If $x + y = 90$, use direct substitution to determine values of x and y which minimize C. Verify your results. [5]

(c) Evaluate the determinant $\begin{vmatrix} 2 & 8 & 5 \\ 1 & 0 & 2 \\ 3 & 4 & 4 \end{vmatrix}$ using cofactor expansion. [6]

(d) A company manufactures stools and tables. Each stool requires 5 hours of carpentry, 4 hours of painting and 35 hours of finishing. Similarly, a table needs 15 hours of carpentry, 4 hours of painting and 20 hours of finishing. During each production period, the carpentry, painting and finishing departments can only work for up to 480 hours, 160 hours and 1190 hours respectively. The company makes $E13$ profit per stool and $E23$ profit per table.
The problem is to determine the number of stools and tables that should be made in order to maximize profits. Formulate this as a linear programming problem. DO NOT SOLVE. [8]

(e) A debt of $E1300$ is to be paid off by payments of $E550$ in 45 days, $E380$ in 100 days and a final payment of $E440$. Interest is at 6.5%

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and the Merchant's rule was used to calculate the final payment. In how many days should the final payment be made?. [5]

(f)

Score	A	B	C	D
1	20	16	18	22
2	25	28	15	21
3	27	20	23	26
4	24	22	23	22

A company has 4 employees $1, 2, 3, 4$ to assign to 4 projects A, B, C, D based on the scores shown in the table above. Determine the assignment schedule that maximizes the total score. [10]

SECTION B

Answer any THREE questions from section B.

B2. (a) Find and classify all stationary points of the function $2x^3 + 2y^3 - 6xy$. [10]

(b) A music producer has been assigned a budget of E60 000 to be spent of advertising and production of a new album. She estimates that spending x thousand Emalangeni in production and y thousand Emalangeni in advertising she can sell $20x^{\frac{2}{3}}y$ albums. If she wants to maximize the sells, how much should she allocate advertising and how much should she allocate to production? [10]

B3. (a) Solve the linear system

$$\begin{array}{rcrcrcr} 2x_1 & + & 6x_2 & + & 4x_3 & = & -2, \\ x_1 & - & 2x_2 & & & = & 4, \\ & & 4x_2 & + & x_3 & = & 2. \end{array}$$

using **Cramer's rule**. [8]

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(b) An economy is based on 3 industries: mining, energy and clothing. Each $E1$ in mining requires $25c$ in mining, $10c$ in energy, and $50c$ in clothing. Each $E1$ in energy takes $40c$ in energy and $20c$ in clothing, while each $E1$ in clothing uses $13c$ in mining and $5c$ in energy. Find the production schedule for the economy if demand is for $E40$ million in mining, $E30$ million in energy, and $E20$ million in clothing. [12]

B4. Two dietary drinks are used to supply protein and carbohydrates. The first drink provides 2 units of protein and 3 units of carbohydrates in each litre. The second drink supplies 5 units of protein and 7 units of carbohydrates in each litre. An athlete requires 12 units of protein and 17 units of carbohydrates. The first drink costs $E42$ per litre and the second costs $E3$ per litre.

(a) The problem is to find the amount of each drink the athlete should consume to minimize the cost and still meet the minimum dietary requirements. Formulate this as a linear programming problem. [8]

(b) Solve the linear programming problem by maximizing the dual. [12]

B5. (a) A stove can be purchased using only one of two options. The first option is to pay $E1300$ cash. The second option requires a down payment of $E600$ followed by payments of $E80$ every month for 12 months. If interest charged is at rate 4.5% compounded monthly, are the two options equivalent? [7]

(b) A farmer wishes to replace his tractor in 4 years time. He figures that he will need $E140\,000$ then. What sum must he invest at the end of each half year in a fund paying 8.5% compounded semi-annually in order to accumulate the price of the tractor? [7]

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(c) How much should you deposit in an account paying 5% compounded quarterly in order to be able to withdraw $E1200$ every 3 months for the next 2 years? [6]

B6. (a) Consider the problem of assigning five jobs $(1, 2, 3, 4, 5)$ to five people (A, B, C, D, E). The assignment costs are given as follows

	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

Determine the optimum assignment schedule. [10]

(b) A computer games internet retailer has four favoured customers who each want a copy of the latest FIFA 2010 computer game. The retailer has one copy available to it at each of the two wholesalers in SA, and can get two further copies, one from each of the two wholesalers in the UK. The cost of each possible allocation of copies (i.e wholesalers) to the customers are

Costs	Customer 1	Customer 2	Customer 3	Customer 4
Wholesaler 1	1	3	6	2
Wholesaler 2	5	2	3	4
Wholesaler 3	9	13	10	8
Wholesaler 4	7	12	8	5

Using the Hungarian Algorithm, find an optimal assignment of wholesalers to customers. [10]

END OF EXAMINATION