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UNIVERSITY OF SWAZILAND

EXAMINATION, 2016/2017

B.Com II, B.Ed (Sec.) II

Title of Paper : Quantitative Techniques

Course Number : MAT202/MS202

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

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SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

A1 (a) Find the total revenue function $R(x, y)$ corresponding to the demand equations

$$p = 7 - x - y$$
$$q = 15 - x - 3y$$

and sketch the corresponding domain. In the above equation definition, p and q are prices and x and y are the corresponding quantities demanded. [6 Marks]

(b) Suppose that an economy consists of two industries, E (electricity producer) and W (water producer) and consider the following assumptions.
Excluding product costs, the demand for electricity in the economy, measured in millions of Emalangeni, is 12, and the demand for water in the economy, also measured in millions of Emalangeni, is 8. The production of E1 of electricity costs the electricity company E0.30 for electricity and E0.10 for water. The production of E1 of water costs the water company E0.20 for electricity and E0.40 for water. How much electricity should the electricity company produce and how much water should the water company produce to meet the external demand?

[8 Marks]

(c) What annual deposit should we be making at an annual rate of 6% in order to have accumulated E80,000 after 17 years? [4 Marks]

(d) i. Use the Method of diagonals to calculate the following determinant [4 Marks]

$$\begin{vmatrix} -5 & -4 & 5 \\ 2 & 0 & -5 \\ 4 & 5 & 2 \end{vmatrix}$$

ii. Reduce the following matrix to *reduced row echelon form* [5 Marks]

$$\begin{bmatrix} -2 & -2 & 0 \\ -6 & 6 & 1 \end{bmatrix}$$

(e) Suppose that the daily demand for butter is given by

$$f(p, q) = \frac{3q}{1 + p^2}$$

and the daily demand for margarine is given by

$$g(p, q) = \frac{2p}{1 + \sqrt{q}}$$

where p and q denote the prices per kilogram (in Emalangeni) of butter and margarine, respectively. Determine where these two commodities are substitute, complementary, or neither.

[4 Marks]

(f) A certain country's production is described by the function

$$f(x, y) = 284x^{1/4}y^{3/4}$$

units, when x units of labour and y units of capital were used

i. What is the marginal productivity of labour and the marginal productivity of capital when the amounts spent on labour and capital are 16 units and 1296 units,respectively?

[7 Marks]

ii. Should the government prioritise capital investment over increased spending on labour at this time in order enhance the country's productivity? [2 Marks]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

B2 TOYCO assembles three types of toys - trains, trucks and cars, using three different operations. The daily limits on the available times for the three operations are 430, 460, and 420 minutes, respectively. The revenues per unit toy train, truck, and car are R3, R2, and R5, respectively. The assembly times per train at the three operations are 1,3, and 1 minutes, respectively. The assembly times per toy train at the three operations are 2,0, and 4 minutes, respectively (a zero time indicates that the operation is not used). The corresponding times per toy car are 1,2, and 0 minutes, respectively.

(a) Let x_1, x_2, and x_3 represent the daily number of units assembled of trains, trucks, and cars, respectively. Formulate the associated Linear Programming Problem (LPP) [6 Marks]

(b) Solve the LPP using the Simplex Method. [14 Marks]

QUESTION B3 [*20 Marks*]

B3 (a) A company has 5 jobs. 5 people applied for these jobs. The company needs to find the minimum salary taken for the jobs. The expected salary of the jobs of person A, B, C, D, E is given below in matrix form:

	Jobs (in thousands)				
Person	Job 1	Job 2	Job 3	Job 4	Job 5
A	4	3	1	5	2
B	7	4	2	10	5
C	8	7	4	6	4
D	3	5	8	7	9
E	5	6	3	8	1

Use the Hungarian method to obtain the optimal allocation under **ALL** the following conditions

i. I. A cannot get job 5.
ii. II. Job 3 gets only C.
iii. III. E cannot get any job.

[10 Marks]

(b) A construction company wants cement at three of its project sites $P1$, $P2$ and $P3$. It procures cement from four plants $C1$, $C2$, $C3$ and $C4$. Transportation costs per ton, capacities and requirements are as follows:

	P1	P2	P3	Capacity(tons)
C1	5	8	12	300
C2	7	6	10	600
C3	13	4	9	700
C4	10	13	11	400
Requirement	700	500	800	

Use the North-West Corner Rule Method and the Stepping Stone method to determine optimal allocation of requirements. [10 Marks]

QUESTION B4 [*20 Marks*]

B4 (a) A company will need to replace a piece of equipment at a cost of R900,000 in 10 years. To have this money available in 10 years' time, a sinking fund is established by making equal monthly deposits into an account paying 4% compounded monthly.

i. How much should each payment be? [4 Marks]

ii. How much interest is earned during the last year? [6 Marks]

(b) The total weekly revenue that a company realizes in producing and selling its products is given by

$$R(x, y) = -3x^2 - 3xy + 623x - y^2 + 701y$$

where x is the number of units of product A and y denotes the number of units of product B produced and sold each week. The weekly cost due to the production of these products is

$$C(x, y) = 50x + 400y + 12000$$

Determine the number of units of product A and product B that the company should produce and sell per week to maximise its profit. [10 Marks]

QUESTION B5 [*20 Marks*]

B5 (a) Use the Gauss-Jordan method to compute the inverse of the following matrix [10 Marks]

$$\begin{bmatrix} 0 & 2 & 1 \\ 2 & 1 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

(b) Solve the system of linear equations

$$\begin{array}{rcrcrcl} -\ x_1 & - & 4x_2 & - & 5x_3 & = & 19 \\ & - & x_2 & - & x_3 & = & 5 \\ -\ 3x_1 & + & 4x_2 & + & 4x_3 & = & -29 \end{array}$$

using the Gaussian elimination method. [10 Marks]

QUESTION B6 [*20 Marks*]

B6 (a) Maximize the function

$$g(x,y) = 1 - xy$$

subject to

$$y - x = 2$$

subject to using the method of Lagrange multipliers. [8 Marks]

(b) An economy has a steel plant, coal mine and transportation.
Each $E1$ worth of steel requires $50c$ from steel plant, $30c$ from coal mine, and $10c$ from transportation. Each $E1$ worth of coal takes $10c$ from steel plant and $20c$ from coal mine and $30c$ from transportation, while each $E1$ worth of transportation uses $10c$ from steel, $40c$ from coal mine, and $5c$ from transportation.
Find the production schedule for the economy if demand is $E2$ million for steel, $E1.5$ million for coal, and $E500,000$ for transportation. [12 Marks]

END OF EXAMIRATION PAPER