UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION, 2016/2017

BASS III, B.Ed (Sec.) III, B.Sc. III, B.Eng. III

Title of Paper : Complex Analysis

Course Number : M313

Time Allowed : Three (3) Hours

Instructions

1. This paper consists of SIX (6) questions in TWO sections.
2. Section A is **COMPULSORY** and is worth 40%. Answer ALL questions in this section.
3. Section B consists of FIVE questions, each worth 20%. Answer ANY THREE (3) questions in this section.
4. Show all your working.
5. Start each new major question (A1, B2 – B6) on a new page and clearly indicate the question number at the top of the page.
6. You can answer questions in any order.
7. Indicate your program next to your student ID.

Special Requirements: NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

SECTION A [*40 Marks*]: ANSWER ALL QUESTIONS

QUESTION A1 [*40 Marks*]

a) Consider the complex number $\phi = 2 - 2i$. Determine the following:

   i) Complex conjugate of ϕ. [1]

   ii) Modulus of ϕ. [1]

   iii) $Im(\phi - \bar{\phi})$ [1]

   iv) Multiplicative inverse of ϕ. [2]

   v) Principal value of the argument of ϕ. [2]

b) Determine the order of each pole of

$$f(z) = \left(\frac{z}{2z+1}\right)^2$$

and the corresponding residue. [3]

c) Find the residue of $f(z) = \frac{z-4}{z^2+1}$ at $z = i$. [3]

d) Express $w = z^2(3 - 3i)$ in the form $w = u(x, y) + iv(x, y)$. [3]

e) Find all values of $\rho = (16)^{1/4}$. [4]

f) Find $Arg(1 - i)$. [3]

g) Determine if $f(z) = \cos(xy) + i(3y - x)$ is analytic at $(0, 0)$. [3]

h) Let C be a positively oriented circle such that $|z| = 4$. Find

$$\int_C \frac{dz}{(z-5)(z+6)}$$

[2]

i) Using the known Maclaurin series for $f(z) = \cos(z)$, find the Maclaurin series of

$$f(z) = \cos(z^3).$$

[2]

j) Using the precise definition of a limit, show that

$$\lim_{z \to 1} \left(\frac{iz}{2}\right) = \frac{i}{2}.$$

[4]

k) Show that when a limit of a function $f(z)$ exists at a point z_0, it is unique. [6]

SECTION B: ANSWER ANY *THREE* QUESTIONS

QUESTION B2 [*20 Marks*]

a) Let C be a positively oriented circle such that $|z| = 4$. Use Cauchy's residue theorem to determine

$$\int_C \frac{(5z-2)dz}{z(z-1)}.$$

[7]

b) Suppose that

$$f(z) = \frac{z}{z^2+1}.$$

Find the residue of f at $z = -i$. [7]

c) Let C be a positively oriented circle such that $|z| = 1$. Find

$$\int_C \frac{(z-1)}{(z+4)(z-7)}dz$$

[6]

QUESTION B3 [*20 Marks*]

a) Let $f(z) = z^2$.

i) Determine if $f(z) = z^2$ is analytic everywhere inside and on a simple closed contour C, $|z| = \pi$ taken in the positive sense. [6]

ii) Using your answer in part i), find

$$\int_C \frac{z^2dz}{z-i}.$$

[7]

b) Suppose that C is a positively oriented circle such that $|z| = 2$. Find

$$\int_C \frac{zdz}{(9-z^2)(z+i)}.$$

[7]

QUESTION B4 [*20 Marks*]

a) Find the Laurent series that represents the function

$$f(z) = \frac{4}{(z-1)(z-2)}$$

in the domain $1 < |z| < 2$. [8]

b) Derive the Maclaurin series for the entire function

$$f(z) = \sin(z).$$

[7]

c) Using your answer in part b), find the Maclaurin series for the entire function

$$f(z) = \sinh(z).$$

[5]

QUESTION B5 [*20 Marks*]

State and prove Cauchy's Integral theorem. [20]

QUESTION B6 [*20 Marks*]

Derive Cauchy-Riemann equations for $f(z) = u(x,y) + iv(x,y)$ at a point (x_0, y_0). [20]

END OF EXAMINATION PAPER