University of Swaziland

Supplementary Examination, July 2013

BSc I, EEng I, BEd I, BASS I

Title of Paper : Algebra, Trig. & Analytic Geom.

Course Number : M111

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Find the *first 4 terms* of the binomial expansion of

$$\left(1-3x^2\right)^{-\frac{2}{3}}. \qquad \text{[6 marks]}$$

(b) Find the *middle term* of the binomial expansion of

$$\left(\frac{1}{\sqrt{p}}-2p^2\right)^{20}. \qquad \text{[4 marks]}$$

(c) Divide

$$\frac{2x^4-3x^3-2x+5}{x^2+1}. \qquad \text{[6 marks]}$$

(d) Evaluate

$$\begin{vmatrix} 4 & -2 & 1 \\ -1 & 1 & 5 \\ 1 & 3 & -1 \end{vmatrix}. \qquad \text{[4 marks]}$$

Question 2

(a) Solve for x.

(i) $\log_3 x^2 = -4$ [2 marks]

(ii) $\log_\pi(4x-19)=0$ [2 marks]

(iii) $\log_2 x+\log_2(x-1)=1$ [5 marks]

(b) Find the equation of the circle that passes through $(2,3)$ and $(-1,6)$ with centre on the line $2x+5y+1=0$. [7 marks]

Hence, find the radius and centre of the circle. [4 marks]

Question 3

(a) Consider the parametric equations

$$x = 1 - 9\sin\theta, \quad y = 2 + 9\cos\theta. \tag{1}$$

i. By eliminating θ, express (1) as a single equation in terms of x and y only. [5 marks]

ii. Fully *describe* the curve defined by (1) and *make a sketch*, showing all the *key features*. [5 marks]

(b) Simplify and express your answers in the form $a + ib$

i. $\left(1 + 2i^{17}\right)\left(1 + 2i^{23}\right)\left(1 + 3i^{25}\right)$ [5 marks]

ii. $\dfrac{25}{3 + 4i} + \dfrac{25}{(1 + 2i)^2}$ [5 marks]

Question 4

(a) Prove

i. $\left(1 - \sin^2\theta\right)\left(1 + \tan^2\theta\right) = 1$ [3 marks]

ii. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ [7 marks]

(b) Solve

$$\begin{aligned} x + y - 2z &= -1 \\ x + \quad z &= 4 \\ x - 2y \quad &= 7 \end{aligned}$$

using Cramer's rule. [10 marks]

Question 5

(a) Given that $z = 2 - 3i$ is a root of

$$z^3 + Az^2 + Bz + 13 = 0,$$

where A and B are real constants, find

(i) the values of A and B [4 marks]

(ii) the 2 other roots [5 marks]

(b) Use synthetic division to work out

$$\frac{x^5 + 2x^3 - 3x - 70}{x - 4}.$$ [4 marks]

(c) Use mathematical induction to prove that

$$P(n) = 1 + 3^{2n-1}$$

is always divisible by 4 (where $n \geqslant 1$ is an integer).

[7 marks]

Question 6

(a) Solve for x

i. $2^{x-\frac{1}{2}} = 4 \cdot 3^x$ [3 marks]

ii. $4^x + 6 = 5 \cdot 2^x$ [5 marks]

(b) Find the value of the sum

i. $40 + 45 + 50 + 55 + \cdots + 5000$ [3 marks]

ii. $2 + 6 + 18 + 54 + \cdots + 39,366$ [3 marks]

(c) Find a solution set of

$$\sin^2\theta - \cos\theta = 0$$

in the interval $-\pi \leqslant \theta \leqslant \pi$. [6 marks]

Question 7

(a) Find the value(s) of x such that the numbers

$$2x-5, x-4, 10-3x$$

form a geometric progression. [5 marks]

(b) Use mathematical induction to prove the formula

$$1+2+2^2+2^3+\cdots+2^{n-1}=2^n-1 \quad n \geqslant 1. \qquad \text{[10 marks]}$$

(c) Simplify

$$x-2\ln e^{x-1}-3\ln e^{1-x}-4\log_4\sqrt{2}+(\cos x+i\sin x)(\cos x-i\sin x).$$

[5 marks]