61

UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2010/11

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	FOUNDATIONS OF MATHEMATICS
COURSE NUMBER	:	M231
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1.1 Write down symbolically, the negation of the statements:

(a) $\exists x, (\neg P(x) \vee Q(x))$; [4]

(b) $\forall x \in \mathbb{R}\, \forall y \in \mathbb{R}\, \exists z \in \mathbb{R}, x^2 + y^2 < z$. [6]

1.2 Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$, where $A \subseteq \mathbb{R}$. Determine the truth set of

$$(\forall y \in A), x + y < 5.$$

[5]

1.3 Determine the truth value in $\mathbb{R}$ of:

(a) $\exists x \in \mathbb{R}$ such that $|x| = -x$; [2]

(b) $\exists x \in \mathbb{R} : x + 4 = x$. [3]

QUESTION 2

2.1 Prove that if $A \Rightarrow B$, $B \Rightarrow C$, and $C \Rightarrow A$, then A is equivalent to B and A is equivalent to C. [10]

2.2 Determine the following sets:

(a) $\{m \in \mathbb{N} : \exists n \in \mathbb{N}$ with $m \leq n\}$; [3]

(b) $\{m \in \mathbb{N} : \forall n \in \mathbb{N}$ we have $m \leq n\}$. [2]

2.3 Let a be an algebraic number and let r be a rational number. Show that $a - r$ is an algebraic number. [5]

QUESTION 3

3.1 State the difference between deductive reasoning and inductive reasoning. Which of the two is a valid form of argument? Explain. [4]

3.2 Steve would like to determine the relative scores of three colleagues in the last M231 test using two facts. First he knows that if Fred did not get the highest score of the three, then Janice did. Second, he knows that if Janice did not get the lowest score, then Maggie got the highest score. Is it possible to get the relative scores of Fred, Maggie and Janice from what Steve knows? If so, who got the highest score and who got the lowest amongst the three? [6]

3.3 Define the following:

(a) Fallacy of affirming the conclusion; [2]

(b) Fallacy of denying the antecedent. [2]

3.4 Using truth tables, analyze the following argument and state whether it is valid or invalid

> "People who work hard get promotions. People who get promotions make more money. Therefore, people who work hard make more money." [6]

QUESTION 4

4.1 Describe a modified induction procedure that could be used to prove statements of the form:

(a) For all integers $n \leq k$, $P(n)$ is true, where $P(n)$ is a statement containing the integer n. [3]

(b) For all integers n, $P(n)$, where $P(n)$ is as in (a). [4]

(c) For every positive odd integer, something happens. [3]

4.2 For all non-negative integers n define the number u_n inductively as

$$\begin{aligned} u_0 &= 0, \\ u_{k+1} &= 3u_k + 3^k \qquad \text{for } k \geq 0. \end{aligned}$$

Prove that $u_n = n3^{n-1}$ for all non-negative integers n. [4]

4.3 If $f(n) = 3^{2n} + 7$, where n is a natural number, show that $f(n+1) - f(n)$ is divisible by 8. Hence prove by induction that $3^{2n} + 7$ is divisible by 8. [6]

QUESTION 5

5.1 Using truth tables, prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. Draw a Venn diagram to illustrate the proof. [10]

5.2 Prove that if $S \subseteq T$ and $T \subseteq \mathbb{R}$, where $S \neq \emptyset$ and $T \neq \emptyset$, and if u is an upper bound for T, then u is an upper bound for S. [3]

5.3 Let $S = \{x \in \mathbb{Q} : x^2 < 2\}$. Prove that $\mathit{inf}\, S = -\sqrt{2}$ and $\mathit{sup}\, S = \sqrt{2}$. [7]

65

QUESTION 6

6.1 (a) What is a partition of a set S? [2]

(b) Let S be a set and let $\mathscr{R}$ be an equivalence relation on S. Prove that the equivalence classes of $\mathscr{R}$ form a partition of S. [10]

6.2 Define a totally ordered set. [2]

6.3 State whether each of the following subsets U of $\mathbb{N}$ is totally ordered or not if the relation on U is "x divides y":

(a) $U = \{24, 2, 6\}$; [2]

(b) $U = \{3, 5, 15\}$; [2]

(c) $U = \{15, 5, 30\}$; [2]

QUESTION 7

7.1 (a) Define the terms maximal element and minimal element of a poset A with a partial order $\mathscr{R}$' [3]

(b) Let the set $B = \{2, 3, 4, 5, 6, 8, 9, 10\}$ be ordered by the relation $\mathscr{R}$ defined by "x is a multiple of y".

i. Show that $\mathscr{R}$ is an order on B. [6]

ii. Find all maximal elements and all minimal elements of B. [4]

iii. Does B have a first and a last element? Support your answer. [2]

7.2 Prove, by contradiction, that if $A \cap B \subseteq C$ and $x \in B$, then $x \notin (A - C)$. [5]

END OF EXAMINATION