UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2009/10

BSc. II

TITLE OF PAPER	:	MATHEMATICS FOR SCIENTISTS
COURSE NUMBER	:	M215
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Find the equation of the straight line through $(-1, 2)$ which is

(i) parallel, (ii) perpendicular to the line $4x + 12y + 3 = 0$ [4]

(b) Describe the solution set of

$$x^2 + y^2 + 2x + 4y + 4 = 0.$$

[3]

(c) Find the unit vector perpendicular to both $\bar{a} = (2, -6, -3)$ and $\bar{b} = (4, 3, -1)$

(i) using a scalar product (ii) using a vector product [5,5]

(d) Derive the vector equation of a straight line passing through two points. [3]

QUESTION 2

(a) Consider a transformation matrix $M(2 \times 2)$. What is the geometrical meaning of $| M |$? [5]

(b) Find the inverse and check the result, or state that inverse does not exist, giving the reason.

(i) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, (ii) $\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 1 & 5 & 2 \end{bmatrix}$. [2, 6]

(c) Solve the following system by Gauss elimination method

$$\begin{aligned} x_1 - x_2 + x_3 &= 0 \\ -x_1 + x_2 - x_3 &= 0 \\ 10x_2 + 25x_3 &= 90 \\ 20x_1 + 10x_2 &= 80 \end{aligned}$$

[7]

QUESTION 3

(a) A ladder 20m long leans against a vertical wall. The bottom of the ladder is pulled away from the wall at the rate of 8m/min. How fast is the top of the ladder moving down the wall when the bottom of the ladder is 12m from the wall? [7]

(b) (i) State, and

(ii) prove the mean value theorem.

[1,4]

(c) Apply the L'Hospital rule to evaluate the following limits

(i) $\lim_{x\to 0^+} \frac{\sin x}{\sqrt{x}}$,

(ii) $\lim_{x\to 0} \frac{\ln \sin^2 x}{\cot x}$.

[3,5]

QUESTION 4

(a) Use the Taylor's series expansion to state and prove

(i) necessary, and

(ii) sufficient conditions theorem for $f(x)$ to have a minimum at x^*.

[4,4]

(b)

(i) Use the quadratic approximation formula to compute $\exp(x)$ for small $|x|$, and estimate the error.

(ii) Use the results from (i) to calculate $\exp(-0.05)$.

[6,3]

(c) Find the third Taylor polynomial of the function

$f(x) = 1 + x^2 + 2x^3$ at $x_0 = 1$. [3]

QUESTION 5

(a)

(i) Define the partial derivatives f_x and f_y of $f(x, y)$ at a point (x_0, y_0).

(ii) Let $f(x, y) = |x|$. Find f_x and f_y at $x = y = 0$.

[3,3]

(b) Let $f(x, y) = 3x^2y$, $x = u + v$ and $y = uv$. Find the partial derivatives f_u and f_v at $u = 2$ and $v = 3$ [7]

(c) Let $f(x, y) = x^3 + y^3 - 3x - 12y + 5$. Find and classify all stationary points.

[7]

QUESTION 6

(a) Apply Lagrange's method to find $min f(X) = x_1^2 + x_2^2$ subject to the constraint

$$2x_1 + x_2 = 2.$$

[8]

(b) Find the area of the region enclosed between the curves

$y = \frac{1}{2}x^2$ and $y = -x^2 + 6$

[4]

(c) Find the volume of a sphere of radius R by using

(i) $V = \int_a^b A(x)dx$.
Where $A(x)$ is an area of the cross section x,

(ii) Formula for the volume of the solid of revolution

[4,4]

QUESTION 7

(a) Find the surface area of a cone using a formula of the area of a surface of revolution.

[6]

(b) Compute the volume under the surface

$$z = f(x, y) = xy + 2$$

over the region D where

$$D = \{x, y : 0 < x < 2, 0 < y < 4\}.$$

[7]

(c) Change to spherical system of coordinates and use a triple integral to find a volume of the sphere.

[7]