University of Swaziland

Supplementary Examination, 2009/2010

BSc II, Bass II, BEd II

Title of Paper : Ordinary Differential Equations
Course Number : M213
Time Allowed : Three (3) hours
Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

Find the general solution of each differential equation.

(a) $y'' - 5y' - 36y = 0$ [5]

(b) $x\mathrm{d}x - e^{x-y}\mathrm{d}y = 0, \quad y(0) = \ln 2$ [10]

(c) $(y'')(y') = \frac{1}{2}$. [5]

Question 2

Use the method of Frobenius to find the series solution of

$$4xy'' + 2y' + y = 0$$

about $x = 0$. [20]

Question 3

(a) Find the inverse Laplace transform of

$$F(s) = \frac{4s + 3}{s(s + 2)^2}.$$ [10]

(b) Solve

$$\mathrm{d}y = (x - 2y)\mathrm{d}x.$$ [10]

Question 4

Solve for $y(x)$:

(a) $y'' + 8y' + 16y = 0$ [5]

(b) $y''' - y'' - 6y' = 6$ [5]

(c) $y'' - 9y = 3e^{9x}, \quad y(0) = 1, \; y'(0) = -1.$ [10]

Question 5

(a) Use Laplace transforms to solve the initial-value problem

$$\ddot{y} + 4\dot{y} + 4y = 0, \quad y(0) = 1, \quad \dot{y}(0) = 1. \qquad [10]$$

(b) Consider the Bernoulli's differential equation

$$y' + yp(x) = y^n q(x).$$

Show that the substitution $v(x) = \left[y(x)\right]^{1-n}$ transforms Bernoulli's equation into a linear equation. [10]

Question 6

Solve

(a) $(x - 2y + 3)\mathrm{d}x - (4y - 2x + 3)\mathrm{d}y = 0$ [10]

(b) $x\,(1 - \ln x + \ln y)\,\mathrm{d}y - y\mathrm{d}x = 0, \quad y(1) = 1$ [10]

Question 7

(a) Find the general solution of

$$\ddot{y} + y = \sec t. \qquad [12]$$

(b) Consider the following statement about first order ODEs.

All separable equations are exact.

Is the statement true or false? Discuss. [8]

Table of Laplace Transforms

$f(t)$	$F(s)$
t^n	$\frac{n}{s^{n+1}}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n}{(s-a)^{n+1}}$
$\frac{1}{a-b}\left(e^{at}-e^{bt}\right)$	$\frac{1}{(s-a)(s-b)}$
$\frac{1}{a-b}\left(ae^{at}-be^{bt}\right)$	$\frac{s}{(s-a)(s-b)}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sin(at)\sinh(at)$	$\frac{2a^2}{s^4+4a^4}$
$\frac{\mathrm{d}^n f}{\mathrm{d}t^n}(t)$	$s^nF(s)-s^{n-1}f(0)-\cdots-f^{(n-1)}(0)$