University of Swaziland

Final Examination, 2009/2010

BSc II, Bass II, BEd II

Title of Paper : Ordinary Differential Equations

Course Number : M213

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

Find the general solution of each differential equation. [1]

(a) $2y'' + 3y' - 2y = 0$ [5]

(b) $xy(1 + y') = 1 + y - x, \quad y(1) = 0$ [10]

(c) $y'' - 4(y')^2 = 0.$ [5]

Question 2

Consider the ODE

$$(x^2 - 1)y'' + 4xy' + 2y = 0.$$

(a) Determine whether $x = 0$ is an ordinary or singular point of the ODE. [5]

(b) Hence find the series solution of the ODE about $x = 0$. Express the solution in closed form. [15]

Question 3

(a) Find the inverse Laplace transform of

$$F(s) = \frac{s + 4}{s^4 + 4s^2}.$$ [8]

(b) Use *two methods* to solve

$$3x^2 + y^2 = 2xyy'.$$ [6, 6]

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[1] Throughout this paper $y' = \frac{dy}{dx}$ and $\dot{y} = \frac{dy}{dt}$.

Question 4

Solve for $y(x)$:

(a) $xy'' + y' = (1 - x)e^{-x}$ [5]

(b) $y^{iv} + 2y''' + y'' - 8y' - 20y = 0$ [5]

(c) $y'' + 4y = 2\cos 4x, \quad y(0) = 1, \; y(\frac{\pi}{4}) = -1$ [10]

Question 5

(a) Use Laplace transforms to solve the initial-value problem

$$\ddot{y} + 4\dot{y} + 13y = 0, \quad y(0) = -1, \quad \dot{y}(0) = 4.$$ [10]

(b) Consider the ordinary differential equation

$$x^2y'' + xy' - 4y = 0.$$

Show that the substitution $x = e^{-2t}$ transforms this equation into

$$\ddot{y} - 16y = 0.$$ [7]

Hence, or otherwise, find the general solution of the equation. [3]

Question 6

Solve

(a) $\dfrac{dy}{dx} = \dfrac{x - y + 1}{x + y + 1}$ [12]

(b) $\left(\dfrac{y}{x} - 2x\right) dx - (2y - \ln x)dy = 0, \quad y(1) = 1$ [8]

Question 7

(a) Use the method of *variation of parameters* to determine the particular integral and hence the general solution of

$$\ddot{y} - 4\dot{y} + 4y = e^{2t}. \qquad [12]$$

(b) Consider the following statement about first order ODEs.

All exact equations can be made linear.

Is the statement true or false? Discuss. [8]

Table of Laplace Transforms

$f(t)$	$F(s)$
t^n	$\dfrac{n}{s^{n+1}}$
$\dfrac{1}{\sqrt{t}}$	$\sqrt{\dfrac{\pi}{s}}$
e^{at}	$\dfrac{1}{s-a}$
$t^n e^{at}$	$\dfrac{n}{(s-a)^{n+1}}$
$\dfrac{1}{a-b}\left(e^{at}-e^{bt}\right)$	$\dfrac{1}{(s-a)(s-b)}$
$\dfrac{1}{a-b}\left(ae^{at}-be^{bt}\right)$	$\dfrac{s}{(s-a)(s-b)}$
$\sin(at)$	$\dfrac{a}{s^2+a^2}$
$\cos(at)$	$\dfrac{s}{s^2+a^2}$
$\sin(at)-at\cos(at)$	$\dfrac{2a^3}{(s^2+a^2)^2}$
$e^{at}\sin(bt)$	$\dfrac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\dfrac{s-a}{(s-a)^2+b^2}$
$\sinh(at)$	$\dfrac{a}{s^2-a^2}$
$\cosh(at)$	$\dfrac{s}{s^2-a^2}$
$\sin(at)\sinh(at)$	$\dfrac{2a^2}{s^4+4a^4}$
$\dfrac{\mathrm{d}^n f}{\mathrm{d}t^n}(t)$	$s^nF(s)-s^{n-1}f(0)-\cdots-f^{(n-1)}(0)$