UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2008/9

TITLE OF PAPER	:	DYNAMICS II
COURSE NUMBER	:	M 355
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Given the following Lagrangian function

$$L = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 - \frac{1}{2}\kappa(x_1^2 + x_2^2) - \frac{1}{2}\kappa(x_2 - x_1)^2$$

for a certain mechanical system.

(i) Find the corresponding Hamiltonian function. [6 marks]

(ii) Using Hamilton's equations obtain the equations of motion for the system. [6 marks]

(b) The Hamiltonian for a system is given by

$$H = \frac{1}{a}p^a$$

where a is a constant. Given that p is a generalized momentum conjugate to the generalized coordinate q, prove that the Lagrangian for the system is given by

$$L = \left(\frac{a-1}{a}\right)\dot{q}^{\frac{a}{a-1}}.$$

[8 marks]

QUESTION 4

(a) Find the path which gives a stationary value for the functional

$$\int_0^1 \left(y + (y')^2\right) dx$$

when

(i) $y(0) = 1, y(1) = 2$

(ii) $y(0) = 1, y'(0) = 1$

(iii) $y(0) = 1, y(1)$ not prescribed. [12 marks]

(b) Show that Euler's equation for the functional

$$I = \int_{x=a}^{b} y\sqrt{1 + (y')^2}dx$$

is given by

$$yy'' = 1 + (y')^2.$$

[8 marks]

QUESTION 5

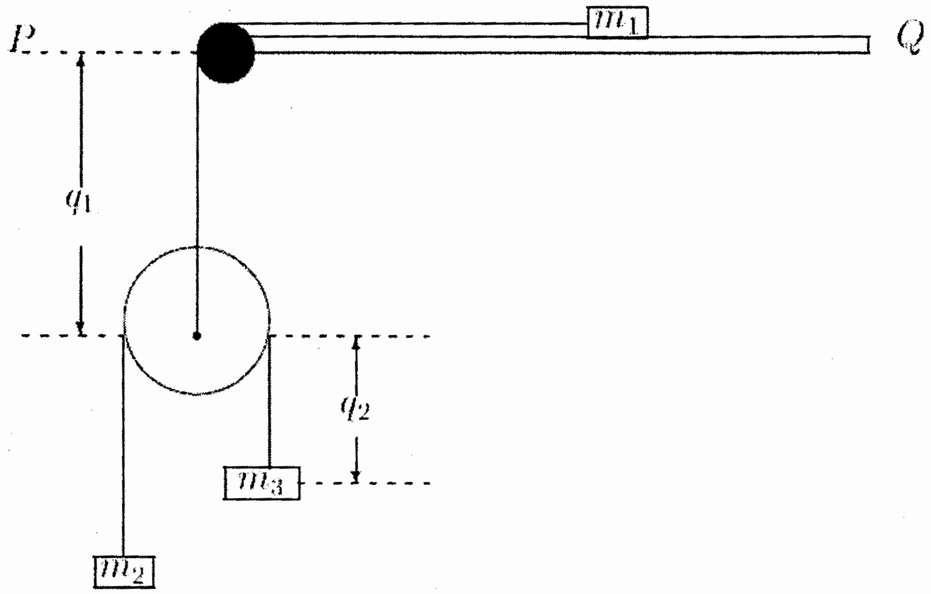
(a) A sytem of two degrees of freedom is described by the Hamiltonian

$$H = q_1p_1 - q_2p_2 - aq_1^2 + bq_2^2$$

where a and b are constants. Show that $F_1 = \frac{p_1 - aq_1}{q_2}$ and $F_2 = q_1q_2$ are constants of motion. [14 marks]

(b) Derive the Hamiltonian for a system with kinetic and potential energy given by $T = \frac{1}{2}m(a^2 + k^2)\dot{\theta}^2$ and $V = mgk\theta$ where θ is the generalised coordinate. [6 marks]

QUESTION 6



Consider the pulley system shown in Figure above with generalized coordinates q_1 and q_2 as indicated. Assume that the pulleys have negligible masses and that PQ is the reference level. Set up the Lagrangian for the system and prove that Lagrange's equations of motion are given by

$$(m_1 + m_2 + m_3)\ddot{q}_1 + (m_3 - m_2)\ddot{q}_2 = (m_3 + m_2)g$$
$$(m_3 - m_2)\ddot{q}_1 + (m_3 + m_2)\ddot{q}_2 = (m_3 - m_2)g.$$

[20 marks]

QUESTION 7

(a) Use the Poisson bracket to show that the transformation

$$q = \sqrt{\frac{P}{\pi\omega}} \sin(2\pi Q) \ , \qquad p = \sqrt{\frac{\omega P}{\pi}} \cos(2\pi Q)$$

is canonical. [6 marks]

(b) Consider the transformation given by

$$Q = q^{\alpha} e^{\beta p} \ , \quad P = q^{\alpha} e^{-\beta p}$$

where α and β are constants. For which values of α and β is this transformation canonical? [5 marks]

(c) Given that,

$$\begin{aligned} A_1 &= \frac{1}{4}(x^2 + p_x^2 - y^2 - p_y^2), \quad A_2 = \frac{1}{2}(xy + p_x p_y) \\ A_3 &= \frac{1}{2}(xp_y - yp_x), \quad A_4 = x^2 + y^2 + p_x^2 + p_y^2 \end{aligned}$$

where x and y are generalized coordinates, and p_x and p_y are the generalized momenta, associates with x and y respectively. Evaluate

(i) $[A_1, A_2]$ [3 marks]

(ii) $[A_3, A_4]$ [3 marks]

(iii) $[A_3, A_2]$ [3 marks]