University of Swaziland

Supplementary Examination – July 2009

BSc I, EEng I, BEd I

Title of Paper : Algebra, Trig. and Analytic Geometry
Course Number : M111
Time Allowed : Three (3) hours
Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Work out

$$\frac{2x^4 + x^2 - 2x - 1}{1 - 2x}. \qquad \text{[8 marks]}$$

(b) Use the binomial theorem to approximate $\sqrt{10}$ correct to 4 decimal places. [7 marks]

(c) Evaluate and simplify

$$\begin{vmatrix} 1 & \tan\alpha & \sec\alpha \\ 0 & \sec\alpha & \tan\alpha \\ -1 & \tan\alpha & \sec\alpha \end{vmatrix}. \qquad \text{[5 marks]}$$

Question 2

(a) Simplify

$$\sin(\theta + 30^0) + \cos(\theta - 60^0). \qquad \text{[5 marks]}$$

(b) Find the exact value of

$$\cos\left(22\tfrac{1}{2}^0\right). \qquad \text{[5 marks]}$$

(c) Find the cube roots of

$$-125i. \qquad \text{[10 marks]}$$

Question 3

(a) Solve

i. $4^x = 5^{2-2x}$. [4 marks]

ii. $10^{\log(2x+7)} = 8$. [6 marks]

(b) Expand and simplify term by term

$$\left(1 - x + \frac{1}{x}\right)^3.$$ [10 marks]

Question 4

(a) Prove by mathematical induction

$$1 + 2 + 2^2 + \cdots + 2^{n-1} = 2^{n+1} - 1, \quad n = 1, 2, \cdots.$$ [10 marks]

(b) Find all values of x in the interval $0 \leq x < 2\pi$, satisfying

$$\sin 2x + \cos x + \cos^2 x = 1 - \sin^2 x.$$ [6 marks]

(c) Evaluate

$$\frac{\left(1 + i\sqrt{3}\right)^4}{\left(-\sqrt{3} - i\right)^3}$$ [4 marks]

and express your answer in the form $a + ib$.

Question 5

(a) Find the 7th term in the expansion of $\left(\frac{2}{x} - x\right)^{-2}$. [4 marks]

(b) Prove

$$\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \tan\theta.$$ [10 marks]

(c) Find the equation of the circle that passes through $(3, 1)$ and $(6, 4)$ with centre on $x + 3y = 5$. [6 marks]

Question 6

(a) Use the rational root theorem and synthetic division to find all real roots of

$$x^3 - 3x^2 + 4 = 0.$$ [10 marks]

(b) Find all roots (real and complex) of

$$z^4 + 5z^2 - 36 = 0.$$ [10 marks]

Question 7

(a) Find the inverse matrix of

$$A = \begin{pmatrix} 2 & -3 & -3 \\ 1 & 3 & 2 \\ 3 & -4 & -1 \end{pmatrix}.$$ [5 marks]

[13 marks]

(b) Evaluate

$$\left(1 - i\right)^{10},$$

and express in the form $a + ib$. [7 marks]