UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2007

BSc. / BEd. / B.A.S.S. III

TITLE OF PAPER	:	NUMERICAL ANALYSIS I
COURSE NUMBER	:	M 311
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Convert the following binary numbers to their decimal equivalent

   i. $(110101.101)_2$ [4 Marks]

   ii. $(111111...1)_2$ [4 Marks]

   (b) Convert the following decimal numbers to their binary equivalent

   i. 12.3125 [4 Marks]

   ii. 0.6 [6 Marks]

QUESTION 2

2. Consider the iterative scheme

$$x_{n+1} = \frac{12}{1 + x_n}$$

   (a) Find the positive fixed point, s, of the scheme. [5 marks]

   (b) Prove that the scheme converges to s for sufficiently close x_0. [7 marks]

   (c) Determine the order and the corresponding asymptotic error constant for this method. [8 marks]

QUESTION 3

3. Consider the function $f(x) = \sqrt{x} - \cos x$.

   (a) Show that $f(x)$ has exactly one root in [0, 1]. [6 marks]

   (b) Use three iterations of the bisection method to find the root for

$$f(x) = \sqrt{x} - \cos x \qquad \text{on} \quad [0, 1]$$

   [8 marks]

   (c) Find the number of iterations needed to approximate a solution to the equation $x^3 + x - 4 = 0$ on the interval [1, 4] to an accuracy of 10^{-3}. [6 marks]

QUESTION 4

4. (a) Use the definition of the derivative at x_0 to show that if h is sufficiently small, then

$$f'(x_0) \approx \frac{1}{h}\Delta f(x_0).$$

Extend this argument to show that

$$f''(x_0) \approx \frac{1}{h^2}\Delta^2 f(x_0).$$

[10 marks]

(b) Given the data

x	$f(x)$
-2	-1
-1	3
0	1
1	-1
2	3

Construct a forward-difference table, and hence deduce the polynomial of degree ≤ 4 that interpolates f at these points. [10 marks]

QUESTION 5

5. (a) For the scheme $x_{n+1} = x_n + c(x_n^2 - 7)$, find the range of values of c for which convergence to the positive fixed point is guaranteed. For what value of c is convergence quadratic? [10 marks]

(b) The positive root of $f(x) = \alpha - \beta x^2 - x$ with α, $\beta > 0$ is sought and the simple iteration

$$x_{n+1} = \alpha - \beta x_n^2$$

is used. Show that convergence will occur for sufficiently close starting value, provided

$$\alpha\beta < \frac{3}{4}$$

[10 marks]

QUESTION 6

6. (a) Evaluate the integral $\int_0^1 xe^{-x}\,dx$ analytically correct to four decimal places. Use the trapezoidal rule with $h = 0.2$ and the Simpson's rule with $h = 0.25$ to compute the same integral. Compare the errors. [10 marks]

   (b) Find the constants c_0, c_1 and x_1 so that the quadrature formula

$$\int_0^1 f(x)\,dx = c_0 f(0) + c_1 f(x_1)$$

   is exact for polynomials of high a degree as possible. [10 marks]

QUESTION 7

7. (a) Factor the matrix

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

   into LU decomposition, and hence solve the linear system

$$\begin{aligned} 2x - y &= 3 \\ -x + 2y - z &= -5 \\ -y + 2z &= 5 \end{aligned}$$

   [12 marks]

   (b) Show that the iteration scheme $x_{n+1} = (1+x_n)^{1/3}$ converges to a solution of $x^3 - x - 1 = 0$ if the initial guess x_0 is between 1 and 2. [8 marks]