UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2007

B.Sc / BEd / B.A.S.S II

TITLE OF PAPER : CALCULUS I

COURSE NUMBER : M211

DURATION : THREE (3) HOURS.

INSTRUCTIONS :

1. THIS PAPER CONSISTS OF **SEVEN** QUESTIONS
2. ANSWER **ANY FIVE** QUESTIONS.

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

a) **Show** that for a parabola, $y = \alpha x^2 + \beta x + \phi$, α, β, ϕ being constants, $\alpha \neq 0$, there is only one critical value, c, and this value is equal to $\frac{-\beta}{2\alpha}$. (3)

b) State and prove **THE MEAN VALUE** THEOREM. (7)

c) Evaluate the following limits

i) $\lim_{x\to\infty} \frac{\ln x}{x}$ (3)

ii) $\lim_{x\to 0}\left(1+\sin\frac{3}{x}\right)^x$ (7)

QUESTION 2

a) Sketch the graph of the following function:

$$f(x) = \frac{x}{x^2 + x - 2}$$

Indicate all *intercepts, extrema, points of inflection* and *asymptotes* where necessary. (10)

b) Use ***Newton-Raphson*** method to estimate the coordinate of intersection of the curves:

$$y_1 = x^3 \text{ and } y_2 = 4x$$

Take your $x_0 = 1.5000$, and stop after the ***fourth iteration***. Keep every value that you use correct to 4 decimal places. (10)

QUESTION 3

a) Use the *disk method* to show that the volume of a sphere of radius a is given by $\frac{4}{3}\pi a^3$. (10)

b) Show that the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$, is $\frac{64\pi\sqrt{2}}{15}$. A sensible sketch of the region referred to here is necessary. (10)

QUESTION 4

a) Show that the circumference of a circle of radius r, is $2\pi r$, if the circle is defined parametrically by

$x = r\cos t$ $\quad$ $y = r\sin t$ $\quad$ $0 \le t \le 2\pi$ (10)

b) Show that the length of the curve: $f(x) = \frac{1}{2}\left(e^x + e^{-x}\right)$ $\quad 0 \le x \le 2$, is approximately 3.63 units. (10)

QUESTION 5

a) Determine if the following sequence converges or diverges. Find the limit if the sequence Converges.

$$a_n = \left(\frac{n+3}{n+1}\right)^n$$

(10)

b) Consider the following sequence.

$$a_n = \sqrt{n^2 + 2n} - n$$

Check if the sequence is

i) Monotonic, and deduce that it is convergent using an appropriate theorem. (4)

ii) Bounded and prove that its *least upper bound* is 1. (6)

QUESTION 6

Discuss the convergence or divergence of each of the following series. State the name of the test used.

a) $\sum_{n=1}^{\infty} \frac{n+5}{n^3 - 2n + 3}$ (5)

b) $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n\sqrt{n}}$ (5)

c) $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{1}{n^2}\right)$ (5)

d) $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{n}{10}\right)^n$ (5)

QUESTION 7

a) Show that the series

$$\sum_{n=1}^{\infty} \frac{x^n}{n}$$

i. Converges absolutely for $|x| < 1$,

ii. Converges conditionally for $x = -1$, and

iii. Diverges for $x = 1$ and for $|x| > 1$

(10)

b) Consider the following power series.

$$\sum_{n=0}^{\infty} \frac{(1+5^n)}{n!} x^n$$

Determine:

i) The centre of convergence, (2)

ii) The radius of convergence, (3)

iii) The interval of convergence of the series. (5)

END OF SUPPLIMENTARY EXAMINATION